Probing TeV Physics in the Nucleon using large scale simulations of Lattice QCD

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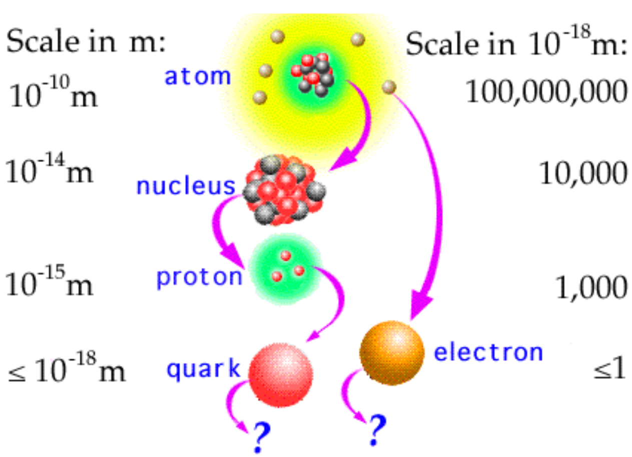

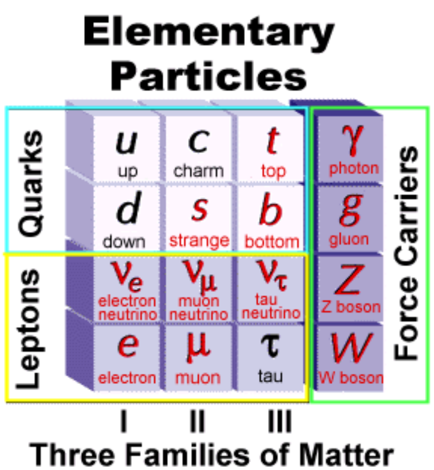

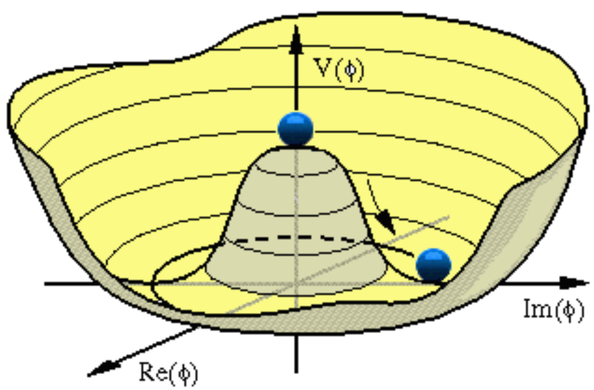

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Oak Ridge: May 24, 2016

Collaboration of Collaborations

PNDME collaboration

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Huey-Wen Lin
- Boram Yoon
- E. Mereghetti
- S. Cohen
- A. Joseph

LHPC collaboration

- Michael Engelhardt
- Jeremy Green
- John Negele
- Andrew Pochinsky
- Sergey Syritsyn

Jlab and W&M collaboration

- Balint Joo
- Kostas Orginos
- David Richards
- Frank Winter

Bhattacharya et al, PRD85 (2012) 054512
Bhattacharya et al, PRD89 (2014) 094502
Bhattacharya et al, PRD92 (2015) 114026
Bhattacharya et al, PRL 115 (2015) 212002
Bhattacharya et al, PRD92 (2015) 094511

Yoon et al, arXiv:1601.07737

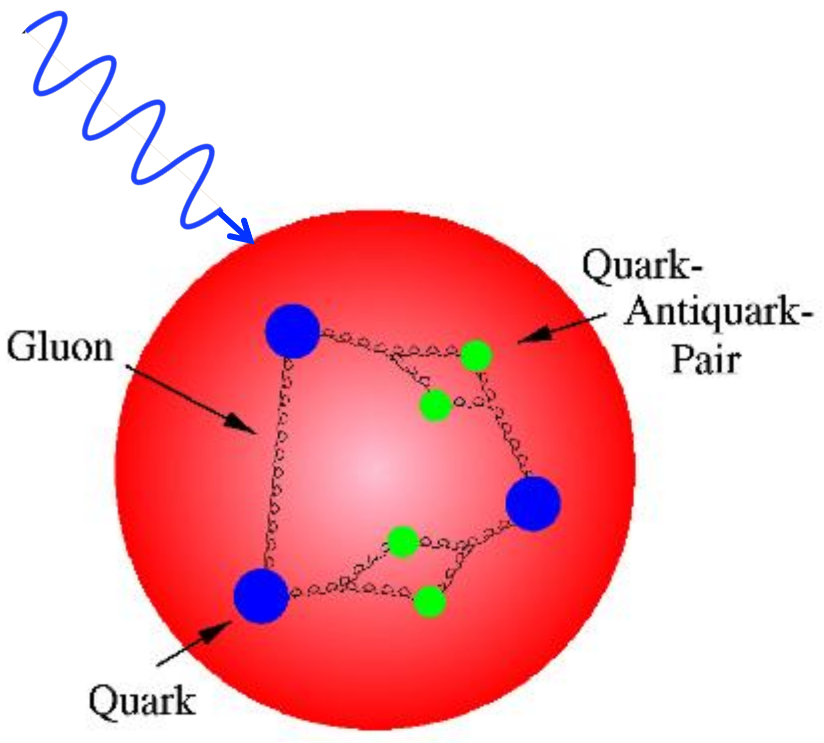
Outline

- Physics Motivation
  - g_A
  - Novel Scalar and Tensor Interactions ($\sim 10^{-3}\, G_F$)
  - Novel Sources of CP violation
- Lattice QCD
- Status of control over systematic errors
- g_A, g_S, g_T
- Neutron Electric Dipole Moment

Why are these calculations interesting and exciting

- Nucleons charges
  - Provide the strength with which nucleons interact
- Form factors
  - How probes at different q^2 couple & see the neutron
  - Size of the nucleon: charge radii
- Neutron Electric Dipole Moment
  - A very sensitive probe of CP violation
  - The universe is observed to be mainly matter
  - Need additional CP violation to explain baryogenesis

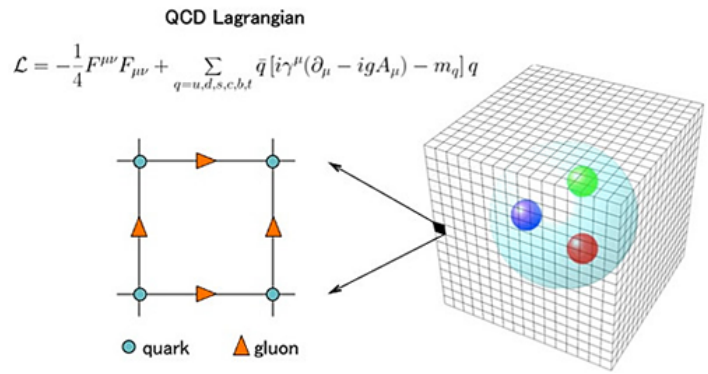
Lattice QCD



- Non-perturbative field theory
- Simulations provide properties of hadrons in terms of quarks and gluons.
- QCD corrections to matrix elements of weak operators

Feynman Path Integral Formulation of QCD

$$Z_{QCD} = \int d\psi\, d\bar{\psi}\, dA\, e^{-S_G - S_F}$$
$$= \int dA\, \det(M)\, e^{-S_G}$$
$$= \int DA\, e^{-S_G + Tr\, Ln\, M}$$



The contribution of the "sea" of each fermion flavor is given by the non-local weight *det(M)* after integration of ψ. It depends only on the gauge field $U_\mu = \exp\{iaA_\mu\}$ and the quark mass *m*.

M is the discretized Dirac Action: a $(3\mathrm{x}4\mathrm{x}L\mathrm{x}L\mathrm{x}L\mathrm{x}T)^2$ sparse matrix

$$S_F = \int d^4x\, \bar{\psi}(x)(i\not{D} + m)\psi(x) \xrightarrow{\textit{lattice}} \sum_{i,j} \bar{\psi}_i M_{ij} \psi_j$$

Physics is extracted from expectation values given by

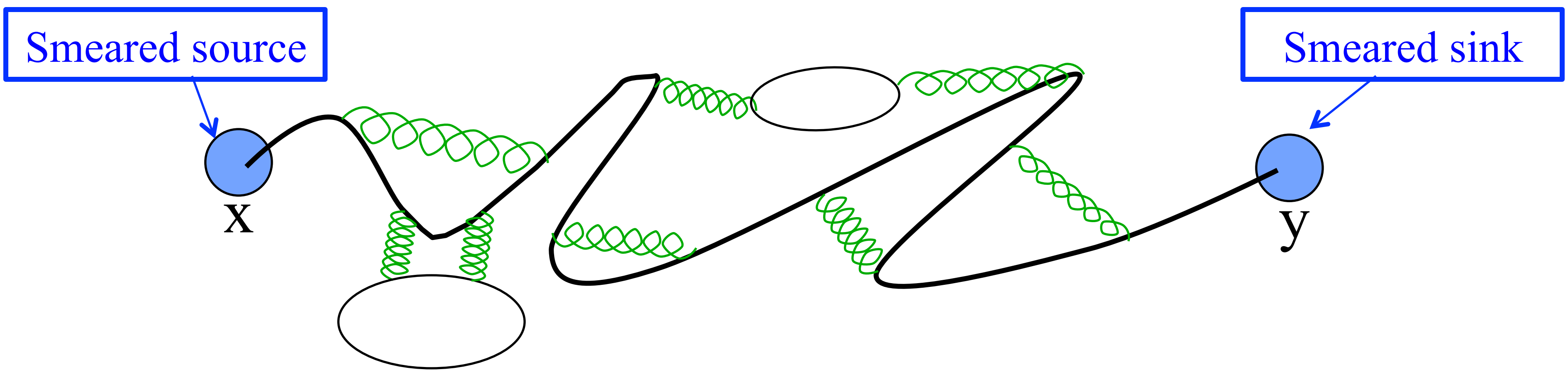
$$\langle \Omega \rangle = \int DA\, \Omega e^{-S_G + Tr\, Ln\, M}$$

What hogs the computer time?

Inversion of the Dirac matrix

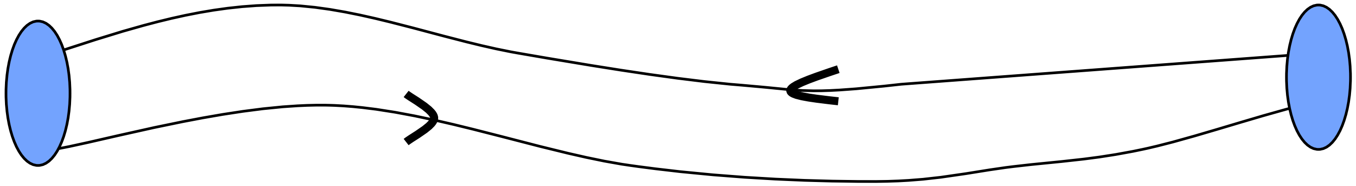
$$S_F(x,y) = \frac{1}{M(x,y)} \text{ where the Dirac Action} = \bar{\psi}(x)M(x,y)\psi(y)$$

$S_F(x,y)$ is the non-perturbative Feynman propagator:
Amplitude of a quark to go from source x to sink y



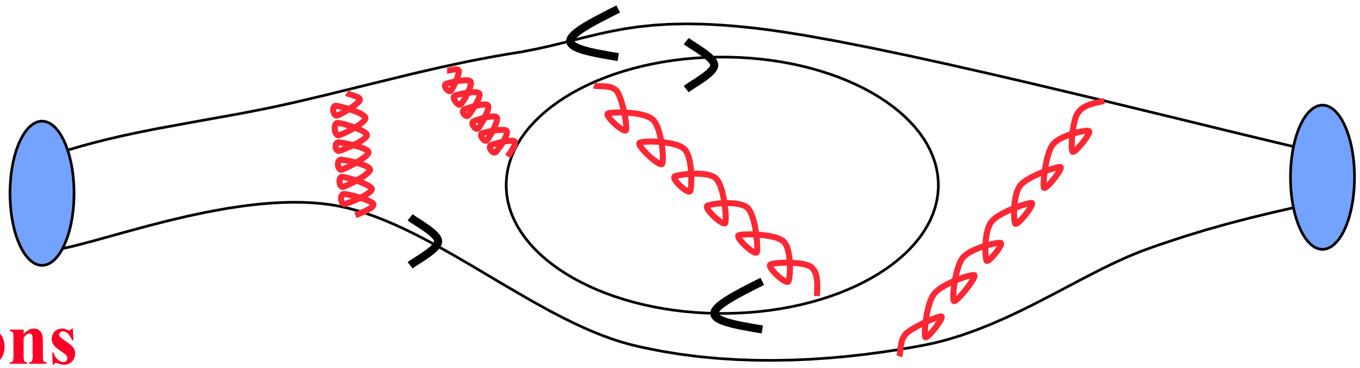
A very useful identity: $S_F(x,y) = \gamma_5 S_F^+(y,x)\gamma_5$

Propagation of stable states in Euclidean time



Pion

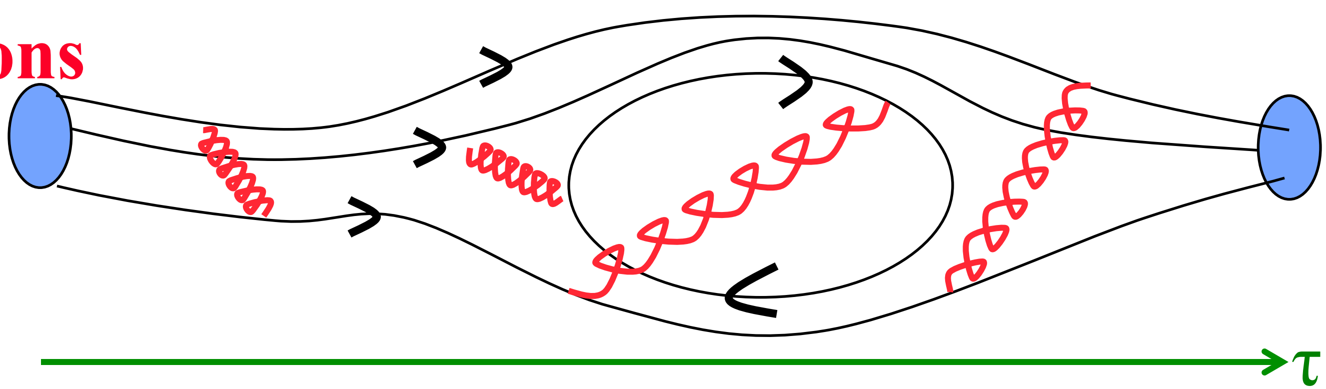
$$S_F(x,y)\gamma_5 S_F(y,x)\gamma_5 = S_F(x,y)\, S_F^{+}(x,y)$$

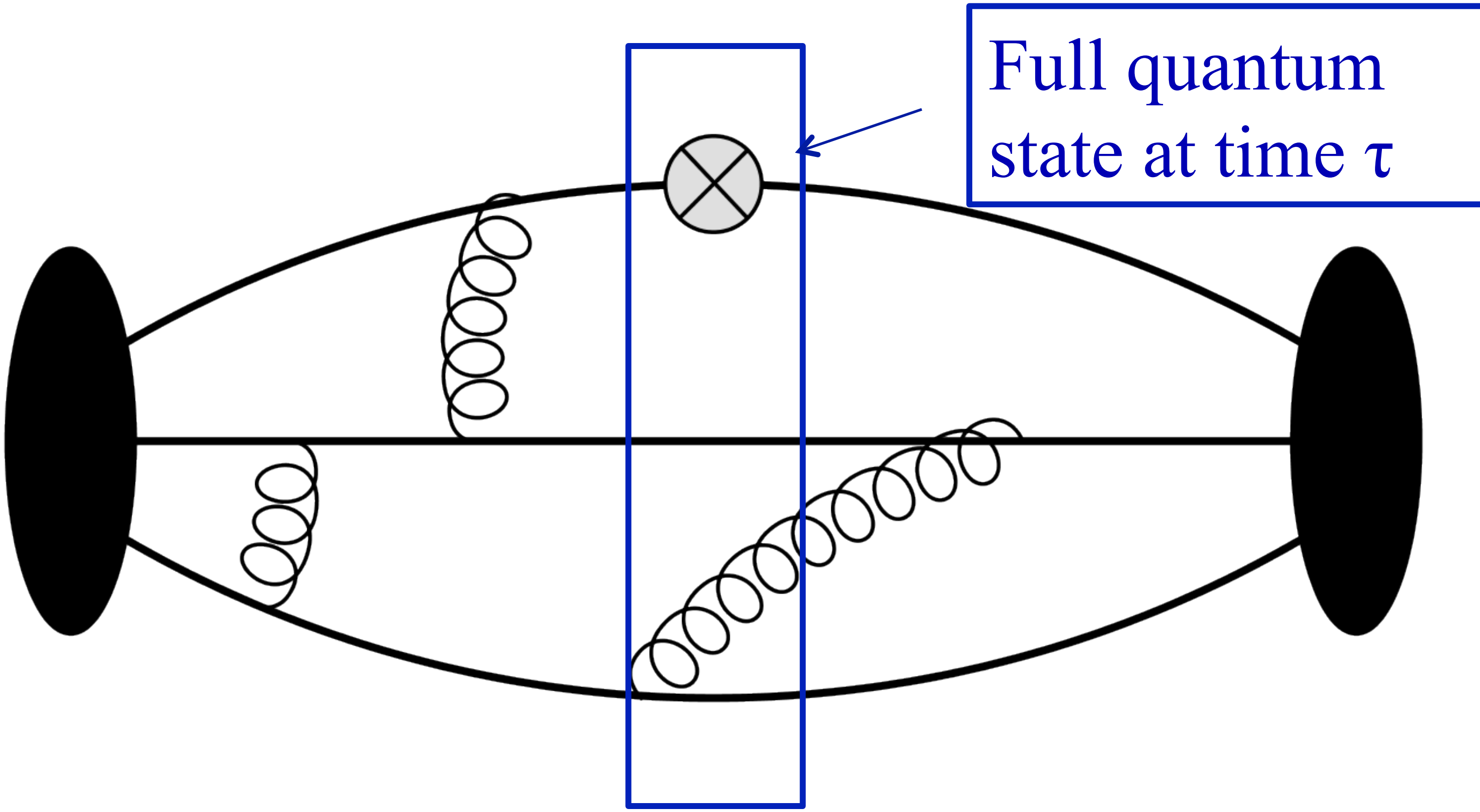


Mesons

$$\langle \pi(\tau)|e^{-L\tau}|\pi(0)\rangle = \sum_n A_n e^{-m_n \tau}$$

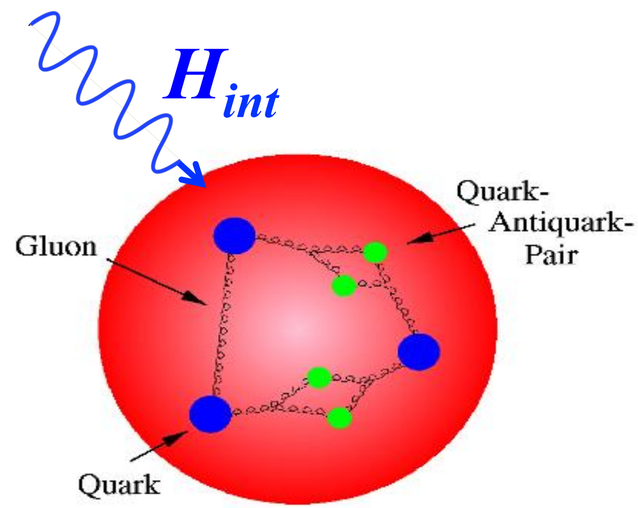
Baryons





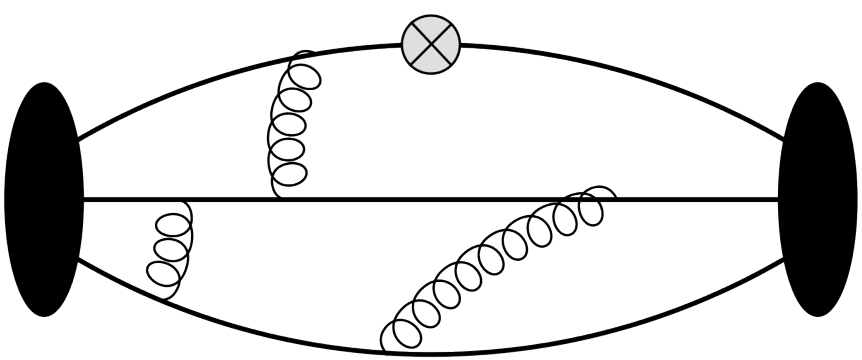
Calculate nucleon 3-point functions

matrix element
<N|H_{int} |N>
in nucleon ground state

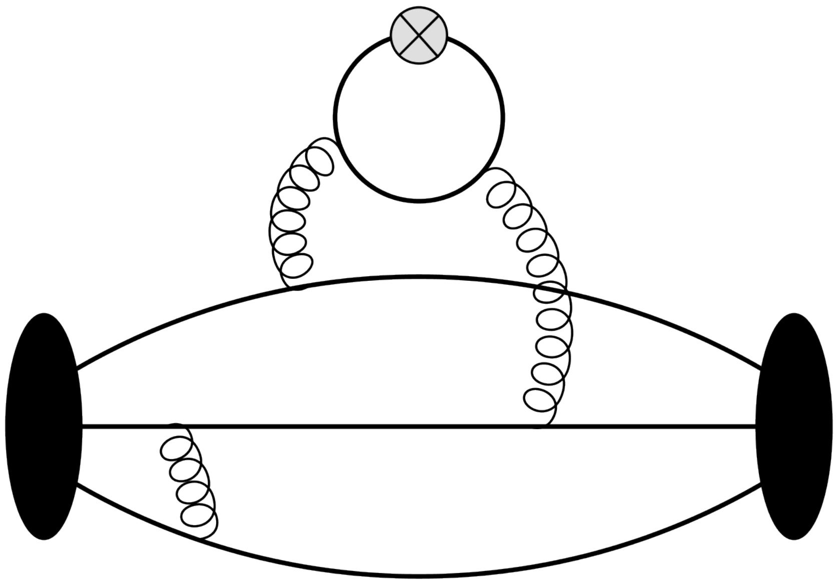


H_{int} is made up of quark bilinears $\bar{u}\gamma_\mu d(\vec{q})$

Need to calculate two types of Feynman diagrams: "connected" and "disconnected"



Connected

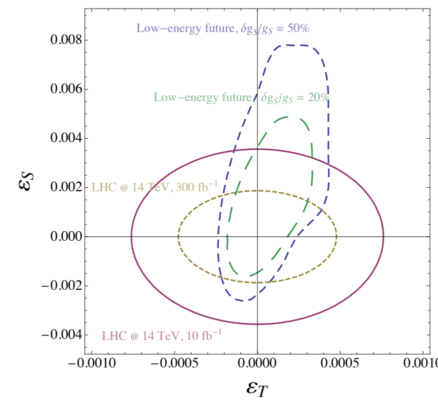


Disconnected

A number of matrix elements within nucleon states become accessible

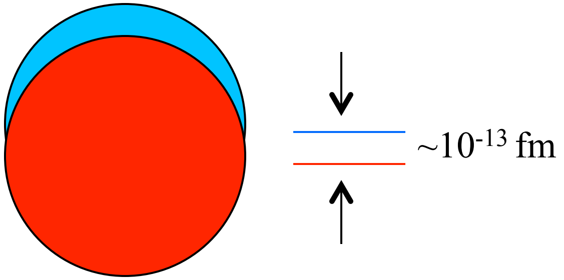
- Iso-vector charges g_A, g_S, g_T $\langle p | \bar{u} \Gamma d | n \rangle$
- Axial vector form factors $\langle p(\vec{q}) | \bar{u} \gamma_\mu \gamma_5 d(\vec{q}) | n(0) \rangle$
- Vector form factors $\langle p(\vec{q}) | \bar{u} \gamma_\mu d(\vec{q}) | n(0) \rangle$
- Flavor diagonal matrix elements $\langle p | \bar{q} q | p \rangle$
- Quark EDM and quark chromo EDM
- Generalized Parton Distribution Functions

I will focus on two quantities



- Iso-vector scalar and tensor charges needed to probe novel scalar and tensor interactions at the TeV scale.

- g_A is the benchmark charge

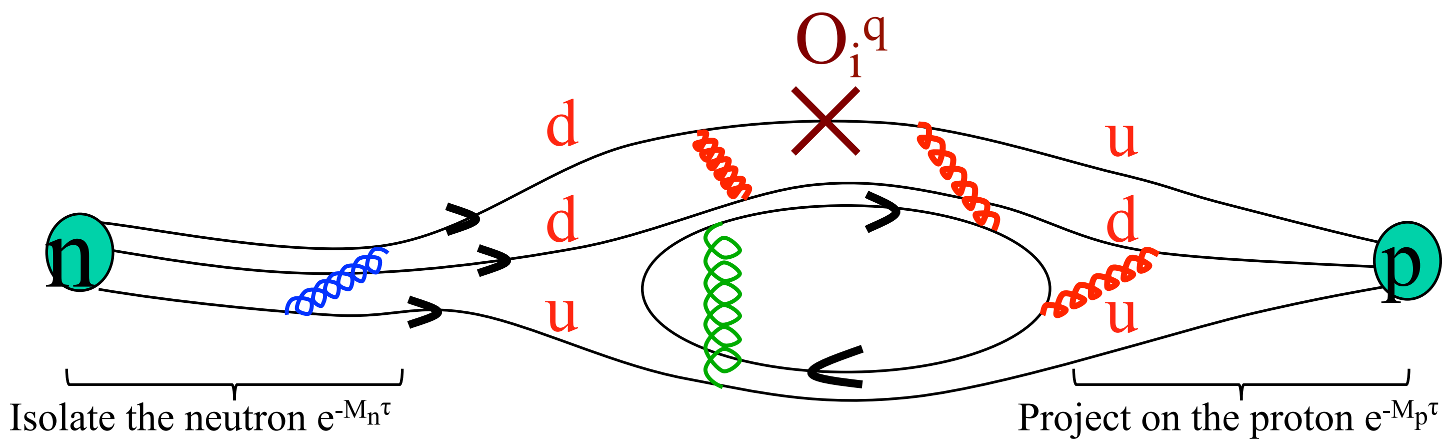


- The contributions of the
  * Θ-term,
  * quark EDM
  * quark chromo EDM
  to the neutron electric dipole moment

Status of numerical simulations being done on Titan at OLCF

As Titan hums
we shake off slumber
sharping our tools
dive into the frigid neutron
in the outpour of numbers
hides the cosmic structure

Achieving <10% uncertainty in nuclear charges $\langle p|\bar{u}\Gamma d|n\rangle$



- **Reached 10% uncertainty in nuclear charges. It required:**
  - High Statistics (O(50,000) – O(128,000) measurements)
  - Demonstrating control over all Systematic Errors:
    - Contamination from excited states
    - Non-perturbative renormalization of bilinear operators (RI_{smom} scheme)
    - ➢ Finite volume effects
    - ➢ Chiral extrapolation to physical m_u and m_d (simulate at physical point)
    - ➢ Extrapolation to the continuum limit (lattice spacing $a \to 0$)

2+1 flavor (u, d, s) clover lattices

m_s “tuned” to its physical value using $M_{s\bar{s}}$

$M_d = m_u$ is lowered to its physical value displaying the dependence of physical quantities on the quark mass

a(fm)	Lattice Volume	M_π L	M_π (MeV)	# of Configs	HP Measure	Low-precision Measure
0.114	$32^3 \times 96$	5.55	316	1000	4x1000	128x1000
0.081	$32^3 \times 64$	4.08	312	1005	3x1005	96x1005
0.080	$48^3 \times 96$	4.09	210	629	4x629	128x629
0.080	64^3 x 128	5.46	210	350	5x350	160x350

AMA analysis on 4 ensembles

$$C_{AMA} = \frac{1}{N_{LP}} \sum C_{LP} + \frac{1}{N_{HP}} \sum (C_{HP} - C_{LP})$$

Blum, Izubuchi, Shintani, PRD88, 094503 (2013)

- LP=Low precision; HP=high-precision
- Cost: 64 LP measurements ≈ 4(HP-LP) bias correction
- 128 well-separated source points on 4 time slices. *(use random source points for smaller correlations compared to fixed source locations)*
- A low cost way to improve statistics (≈16x)

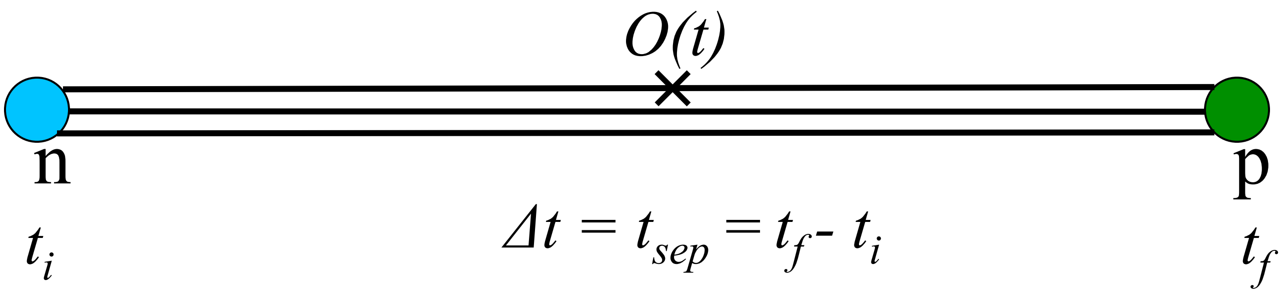
Next level of precision requires:
O(2000) configurations with
O(100) AMA measurements on each.

Reducing excited state contamination: 3-pt fn.

Assuming 1 excited state, the 3-point function is given by

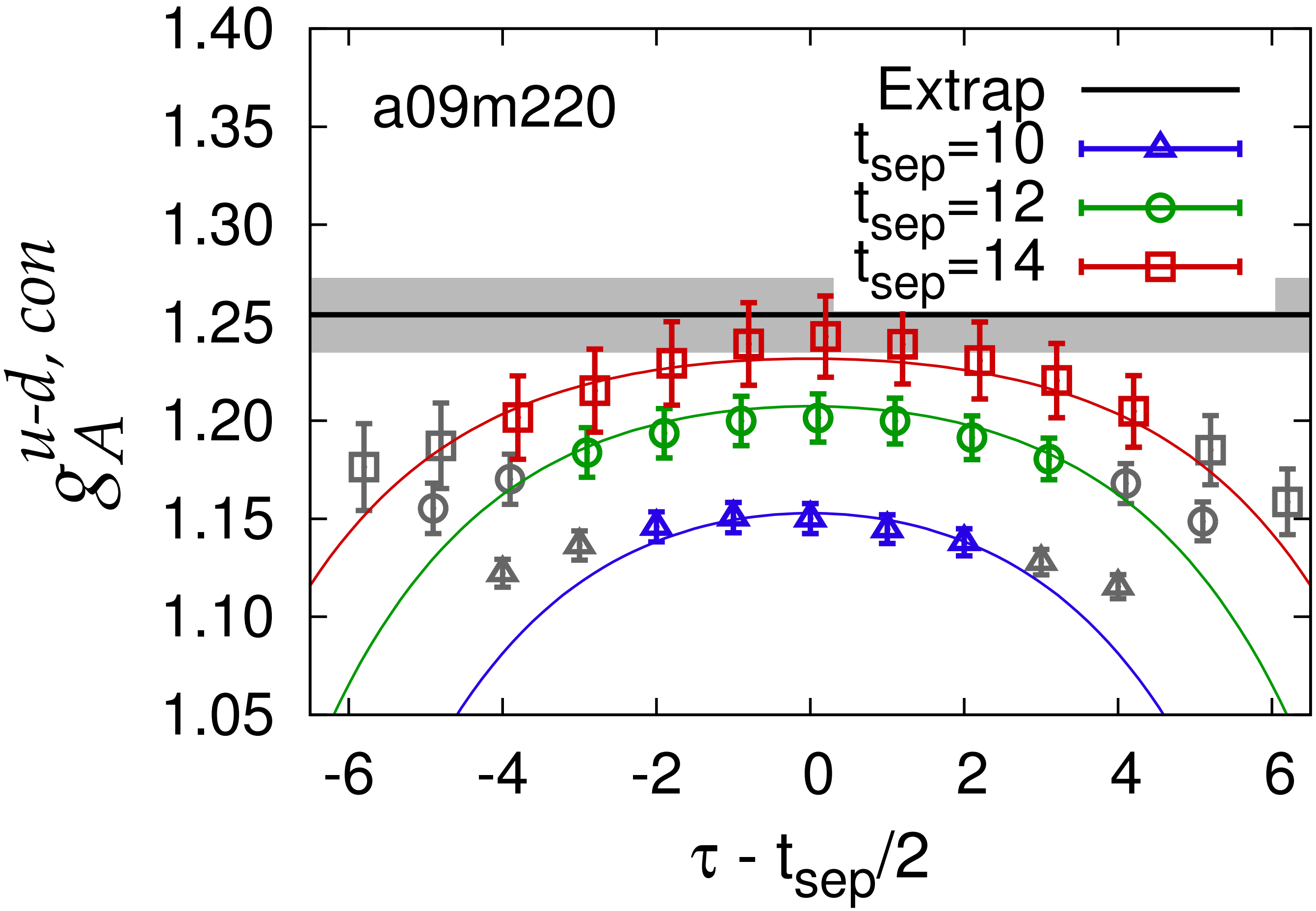
$$\Gamma^3(t_f,t,t_i) = |A_0|^2 \langle 0|O|0\rangle e^{-M_0\Delta t} + |A_1|^2 \langle 1|O|1\rangle e^{-M_0\Delta t} e^{-\Delta M\,\Delta t} + A_0 A_1^* \langle 0|O|1\rangle e^{-M_0\Delta t} e^{-\Delta M(\Delta t - t)} + A_0^* A_1 \langle 1|O|0\rangle e^{-\Delta M\, t} e^{-M_0\Delta t}$$

Where M_0 and M_1 are the masses of the ground & excited state and A_0 and A_1 are the corresponding amplitudes.

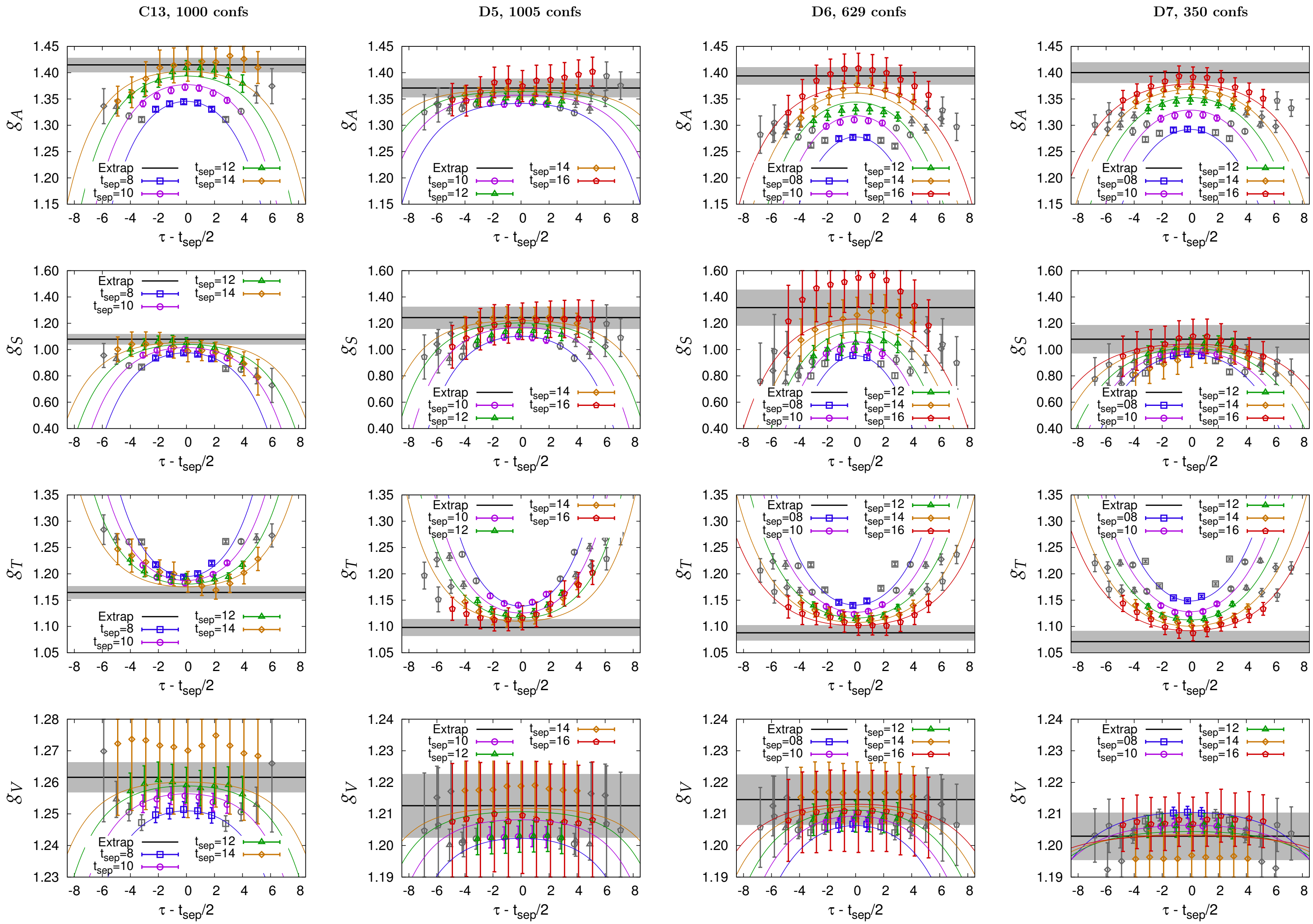


Make a simultaneous fit to data at multiple $\Delta t = t_{sep} = t_f - t_i$

Manifestation of excited state contamination

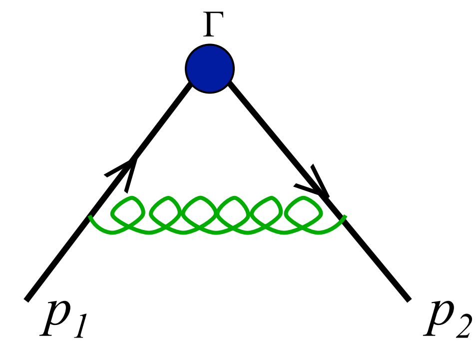


Signal for the charges



Renormalization of bilinear operators

- Non-perturbative renormalization factors Z_Γ using the RI-sMOM scheme ($p_1^2 = p_2^2 = q^2$)
  - Need quark propagator in momentum space
- Need to demonstrate: there exists a window
  $$\Lambda_{QCD} << p << \pi/a$$
- Smearing introduces artifacts
  - Gluon momentum above ($\sim 1/a$) are averaged out
  - May shrink the $\Lambda_{QCD} << p << \pi/a$ window
- Renormalized results presented in $\overline{\mathrm{MS}}$@2 GeV:
  - 2-loop matching
  - 3-loop running to 2 GeV



Martinelli et al, (1995) hep-lat/9411010
Strum et al, (2009) hep-ph 0901.2599

Analyzing lattice data $\Omega(a, M_\pi, M_\pi L)$:

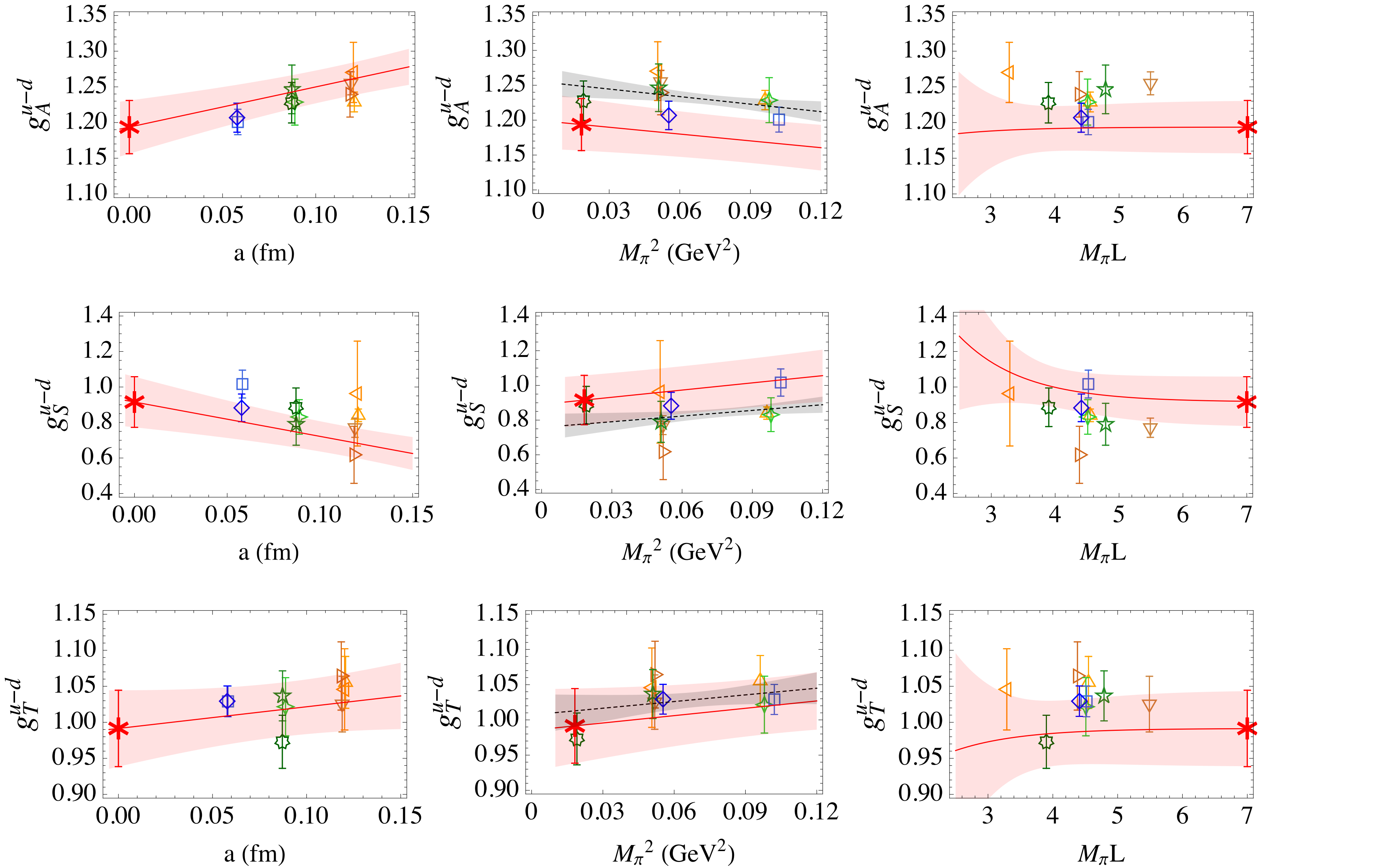
Extrapolations in $a, M_\pi^2, M_\pi L$

We use lowest order corrections when fitting lattice data w.r.t.

- Lattice spacing: a
- Dependence on light quark mass: $m_q \sim M_\pi^2$
- Finite volume: $M_\pi L$

$$g(a, M_\pi, L) = g + A\, a + B\, M_\pi^2 + C\, e^{-M_\pi L} + \ldots$$

AMA: Simultaneous extrapolation in a, M_π^2, $M_\pi L$

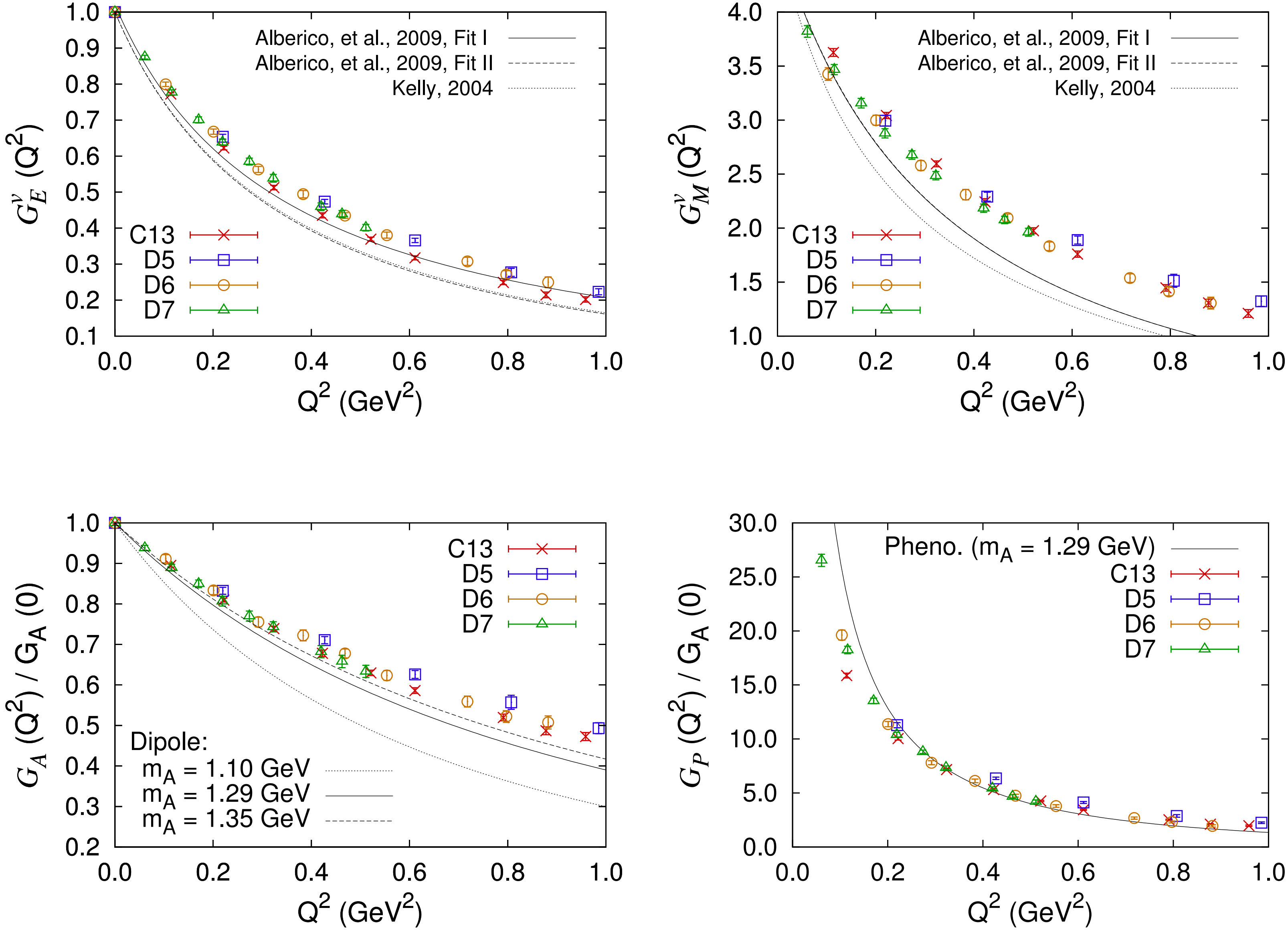


Status of results on iso-vector charges (preliminary– to appear June, 2016)

**	$\mathbf{g_A = 1.23}$
**	$\mathbf{g_S = 1.02}$
***	$\mathbf{g_T = 1.01}$

Expt. $g_A = 1.276(3)$

Vector and Axial form-factors



What is needed for obtaining isovector charges with 2% total errors?

2000 lattices (10–15K trajectories) with
large volume $M_\pi L \geq 4$

$a = 0.1,\ 0.075,\ 0.05$ fm
$M_\pi = 300,\ 200,\ 140$ MeV

O(200,000) measurements

This is attainable by 2020

What is needed for obtaining isovector charges with 2% total errors?

2000 lattices (10–15K trajectories) with
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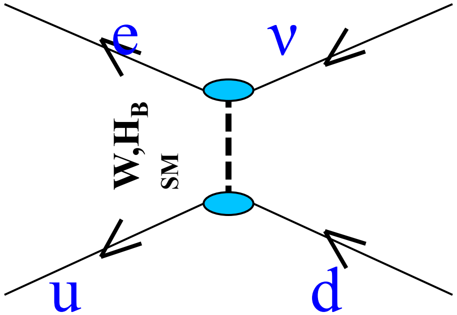
$a = 0.1, \; 0.08, \; 0.05$ fm
$M_\pi = 300, \; 200, \; 140$ MeV

O(200,000) measurements

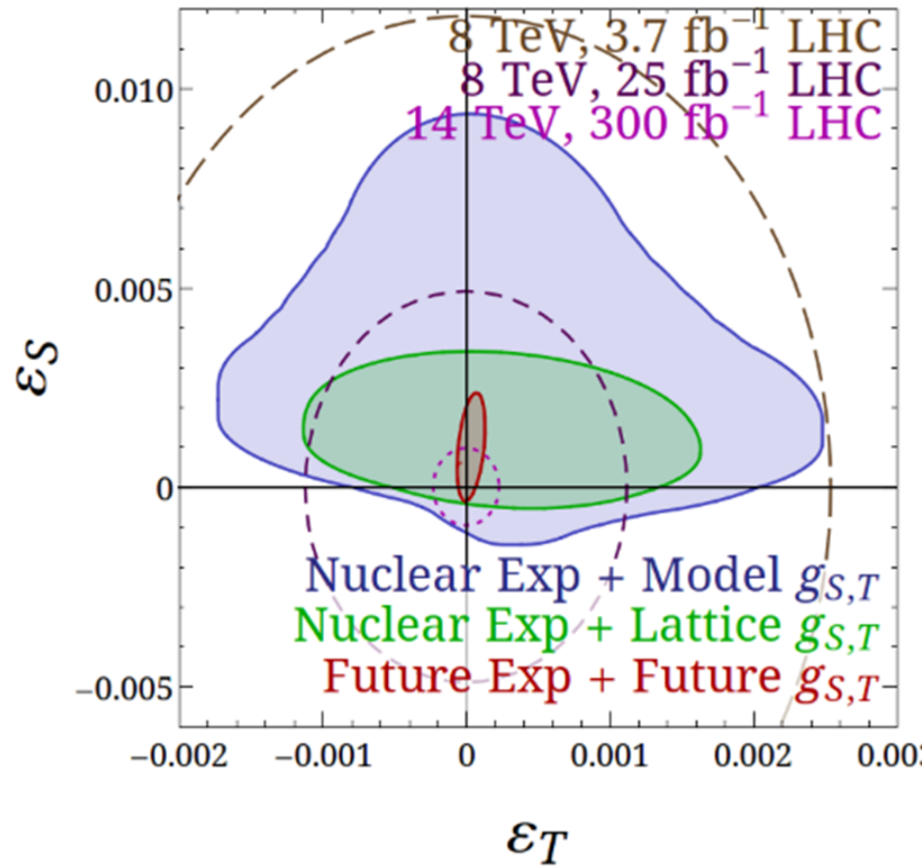
This is attainable by 2020

Constraints on $[\varepsilon_S, \varepsilon_T]$: β-decay versus LHC



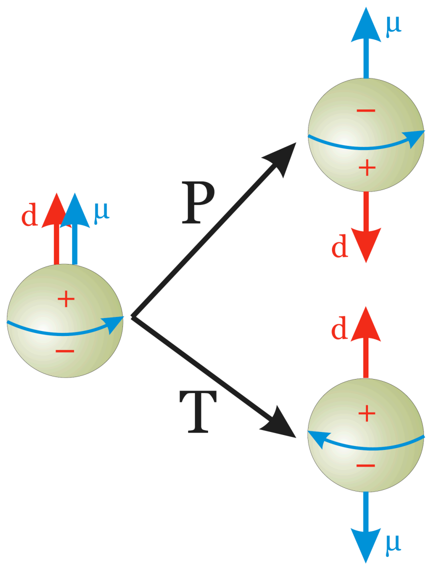
LHC

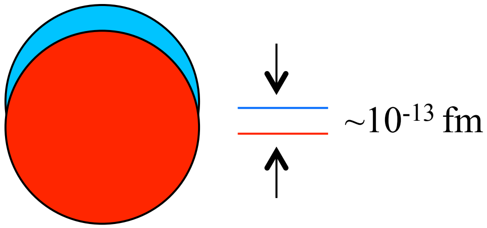
- LHC can provide constraints comparable to low-energy ones with $\delta g_S/g_S$ ~10% and b, $b_\nu \sim 10^{-3}$
- LHC @ 14 TeV and 300 fb^{-1}: look for events with an electron and missing energy at high transverse mass

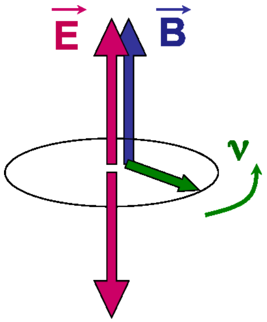


Neutron Electric Dipole Moment

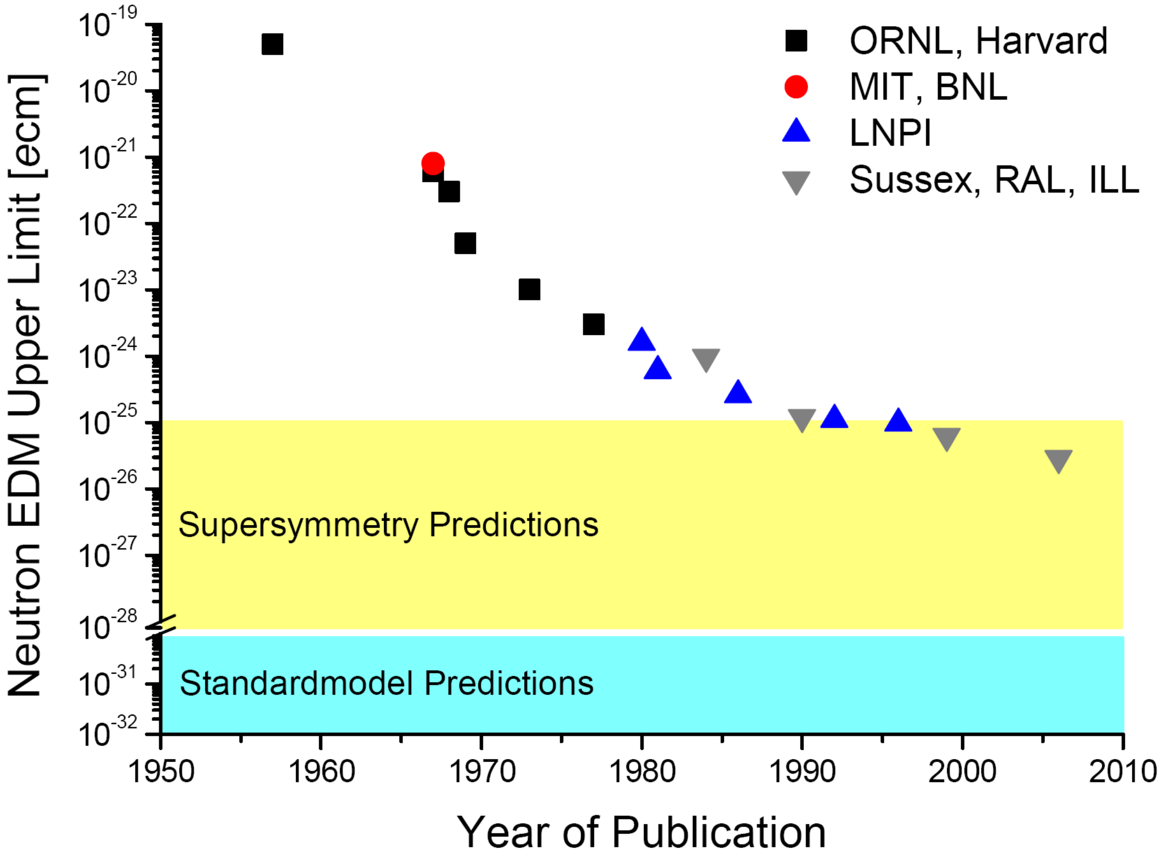
- New (larger) CP violation needed to explain weak scale Baryogenesis
- All CPV interactions contribute to the nEDM
- nEDM provides stringent constraints on BSM theories
- Need precise values of matrix elements of CP violating effective operators to convert bounds on nEDM into bounds on BSM parameters.







nEDM: History & Promise

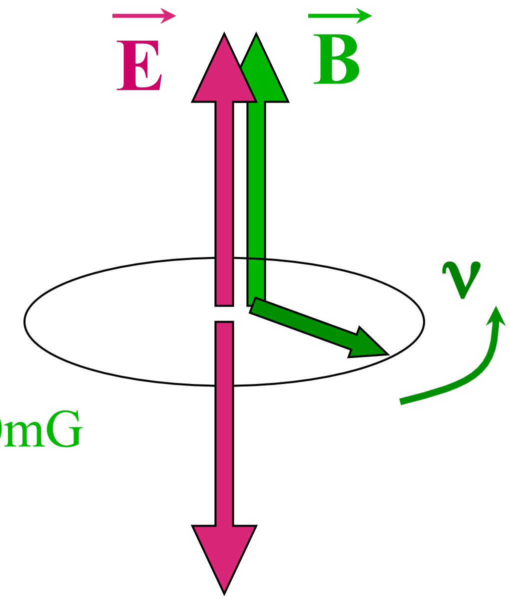


Future:
UCN experimental sensitivity: $S \propto E\sqrt{N\tau}$

- ORNL
- Sussex
- ILL
- PSI
- Munich
- TRIUMF
- LANL
- …

$d_n \leq 2.9 \times 10^{-26}$ e cm

Principle of the Measurement



For B ~ 10mG
ν = 30 Hz

$$\nu = -[2\mu_n B \pm 2d_n E]/h$$

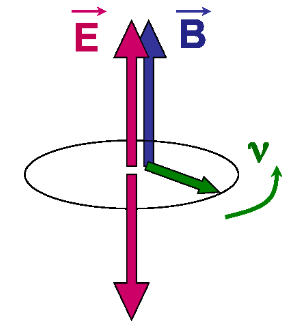
$$\Delta\nu = -4d_n E/h$$

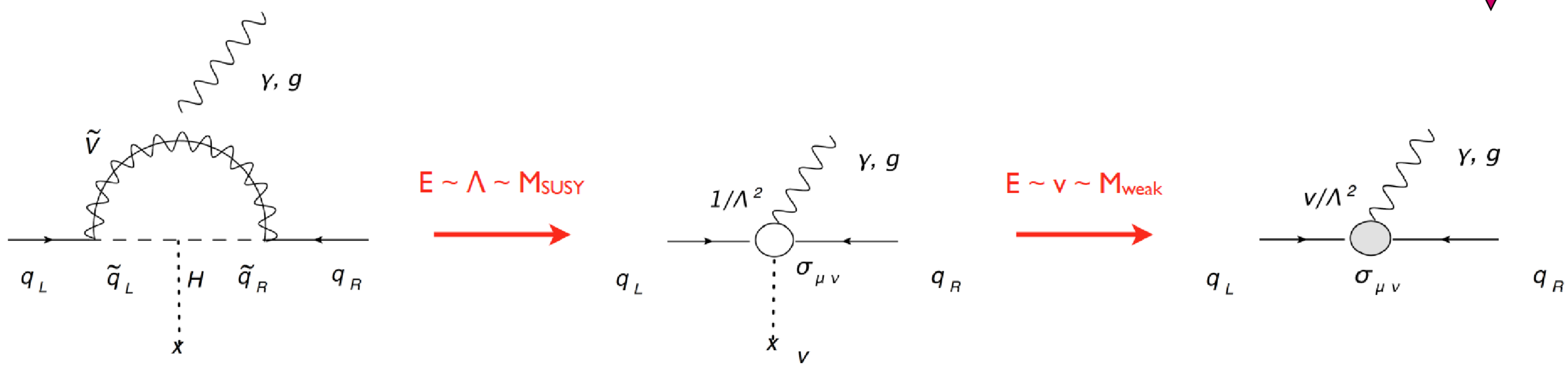
$$d_n = h\frac{\Delta\nu}{4E}$$

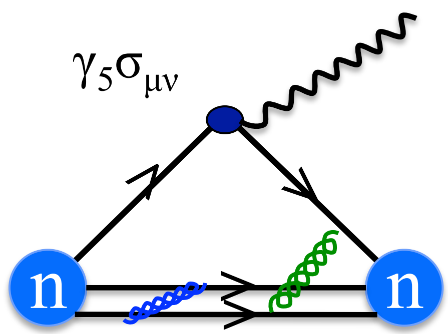
For E = 50kV/cm
d_n= 4x10^{-27}e·cm
Δν = 0.2 μHz
(~60 days for 2π rotation)

Sensitivity of Ramsey Method $S \propto E\sqrt{N\tau}$

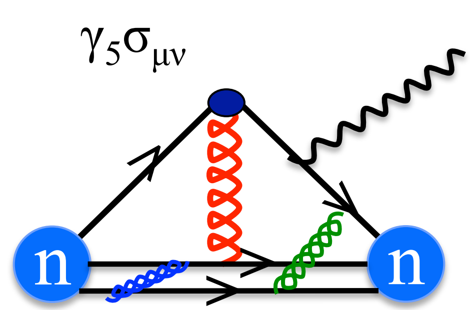
Novel CP violation: operators in EFT







γ attaches to the vertex



- 4-pt function as γ can attach to any quark line
- Gluon free end can attach to any quark line

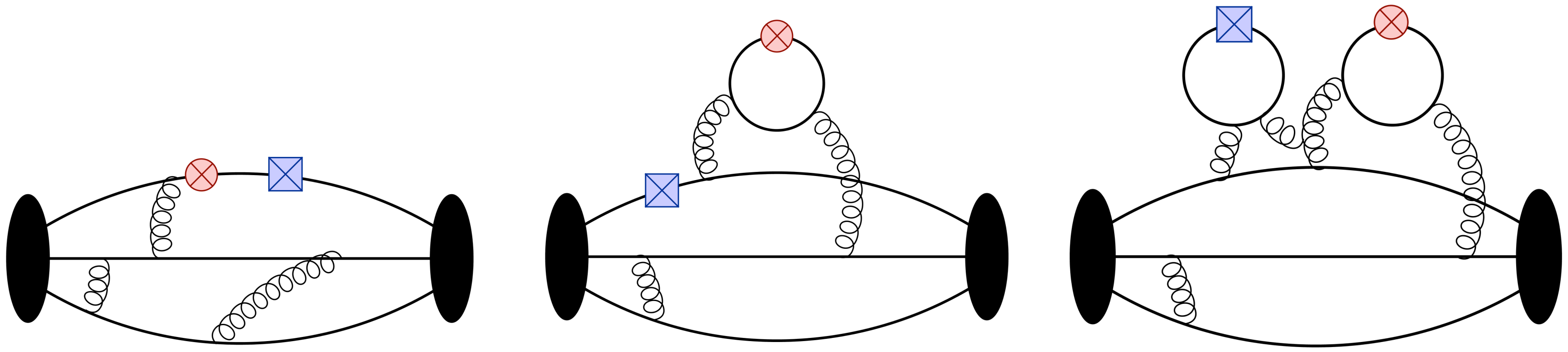
Quark-EDM

$$\bar{q}\,\sigma_{\mu\nu}\gamma_5\, q\; F^{\mu\nu}$$

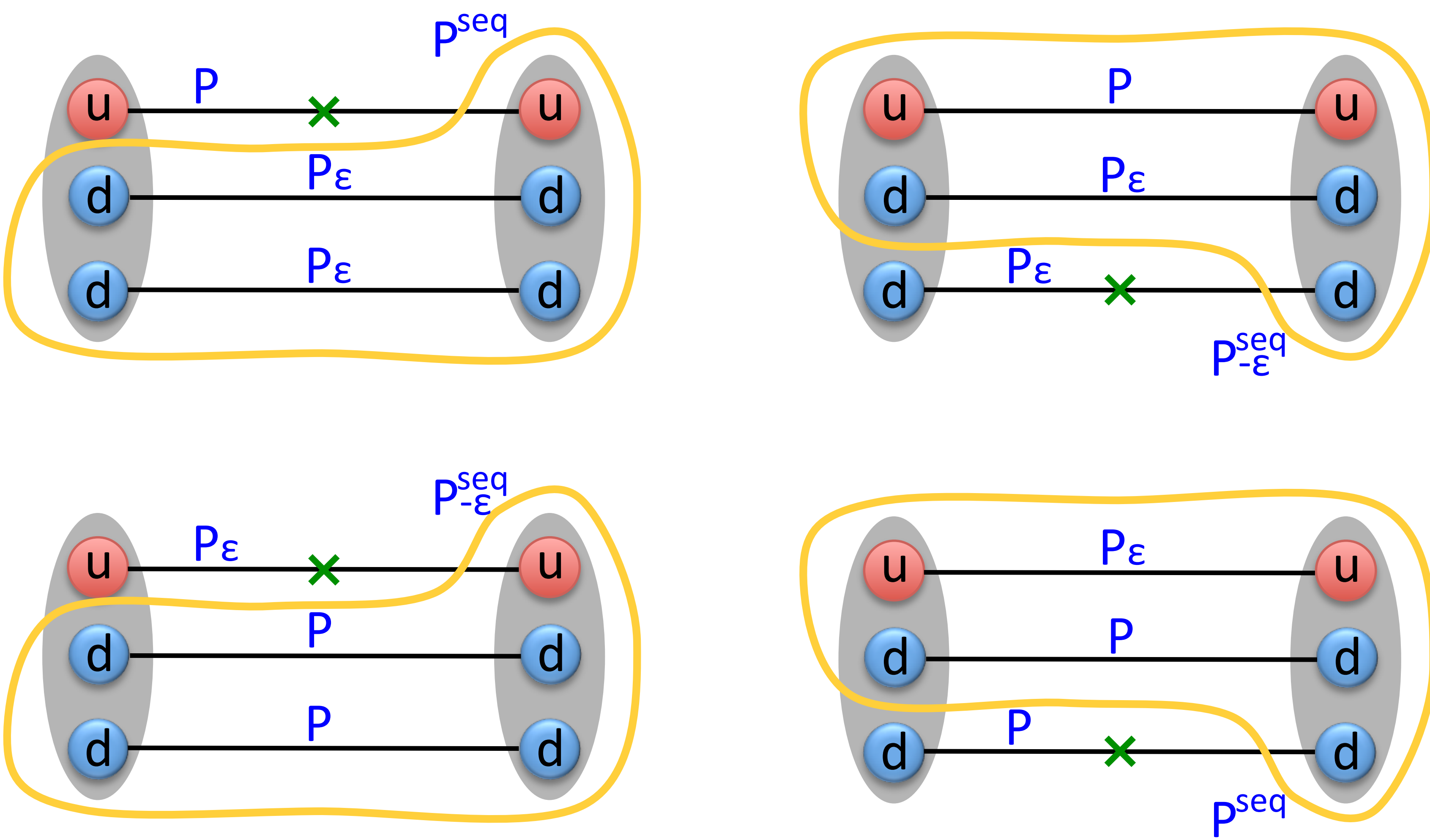
Chromo-EDM

$$\bar{q}\,\sigma_{\mu\nu}\gamma_5\, q\; \lambda^a G_a^{\mu\nu}$$

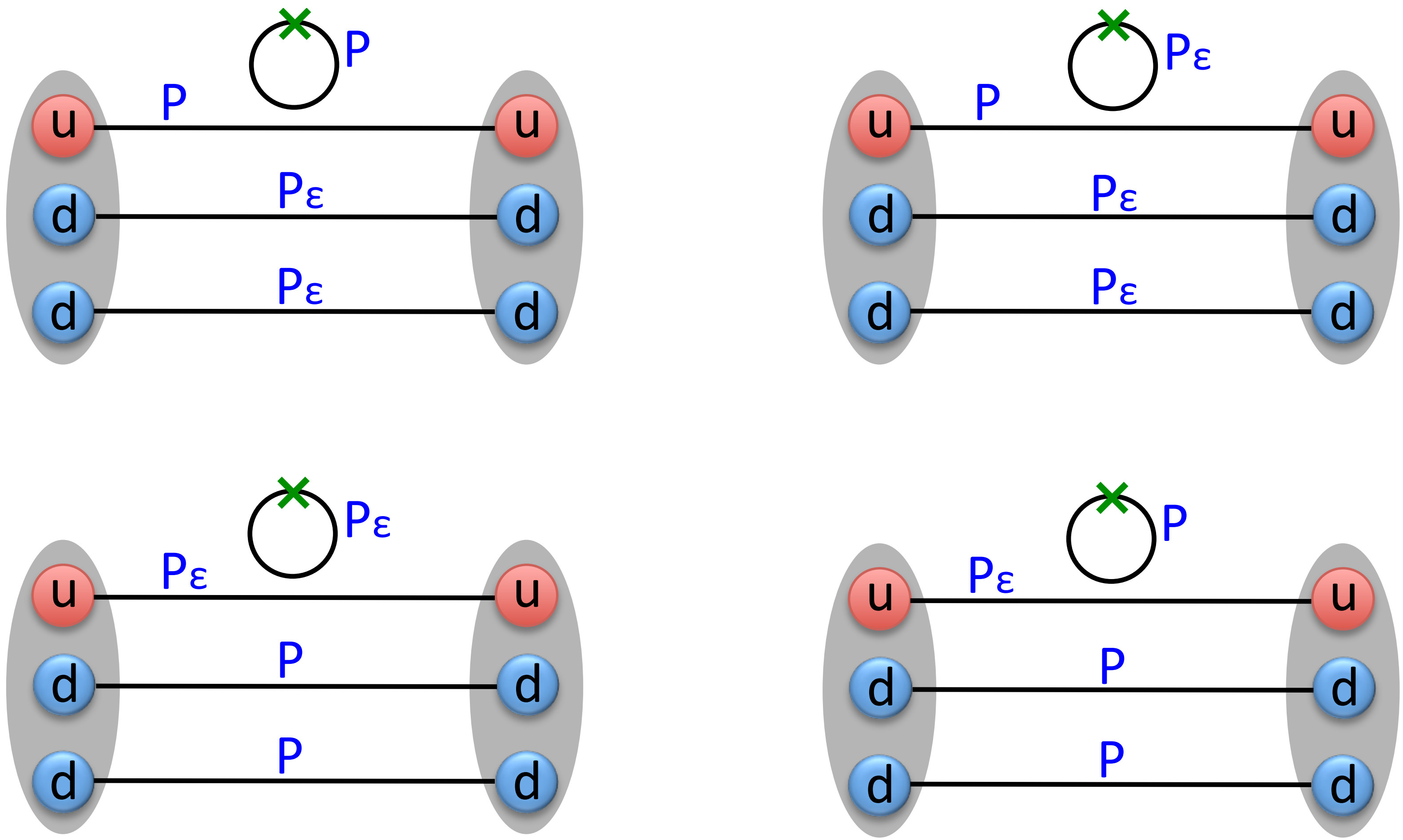
Quark Chromo EDM: 4-pt functions



Connected Contribution



Disconnected Contribution

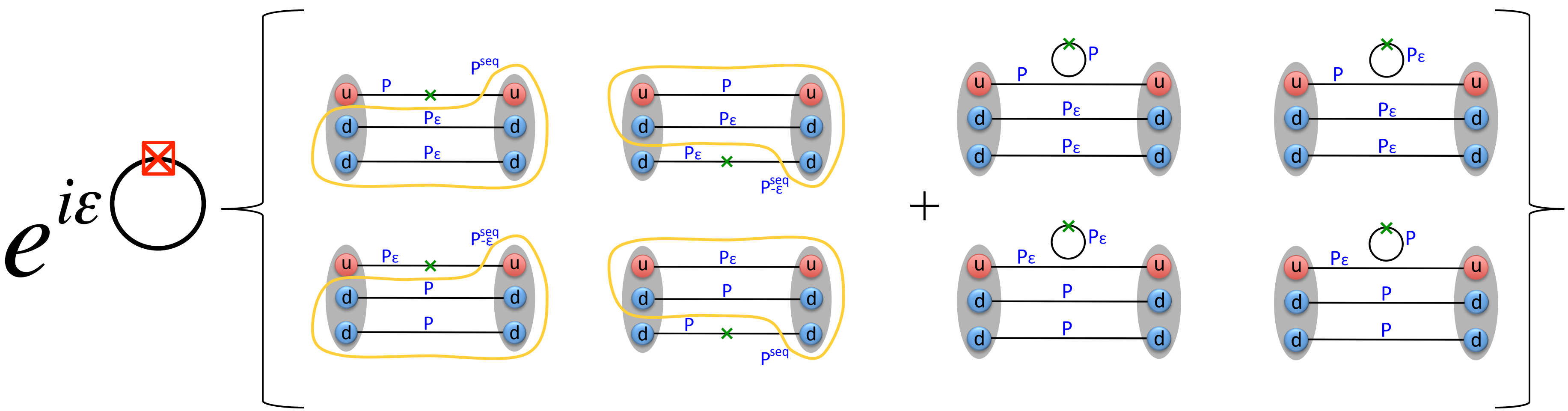


Reweight by the ratio of determinants

$$\frac{Det[\not{D}+m-\frac{r}{2}D^2+\Sigma^{\mu\nu}(c_{SW}G_{\mu\nu}+i\varepsilon\tilde{G}_{\mu\nu})]}{Det[\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu}]}$$

$$=\exp\{Tr\ Ln[1+i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}]\}$$

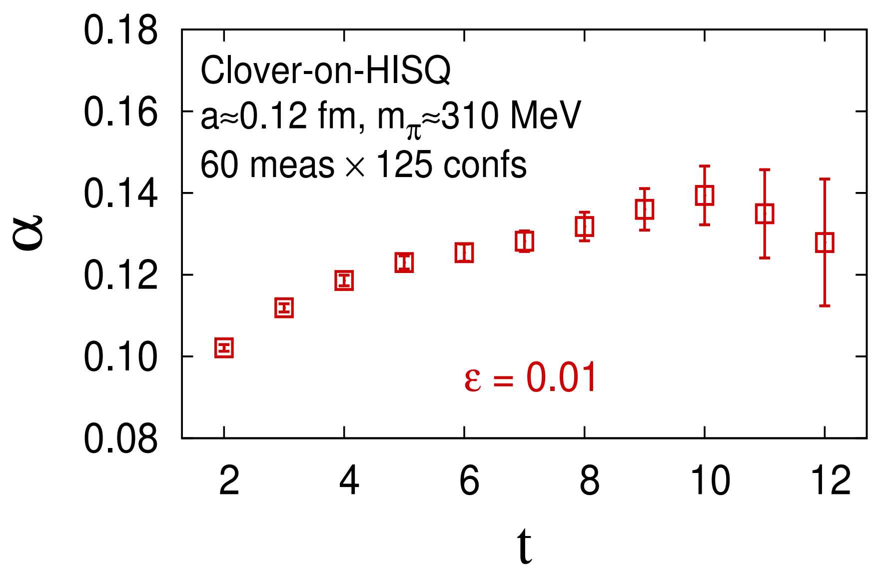
$$\approx\exp\{Tr\ i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\}$$

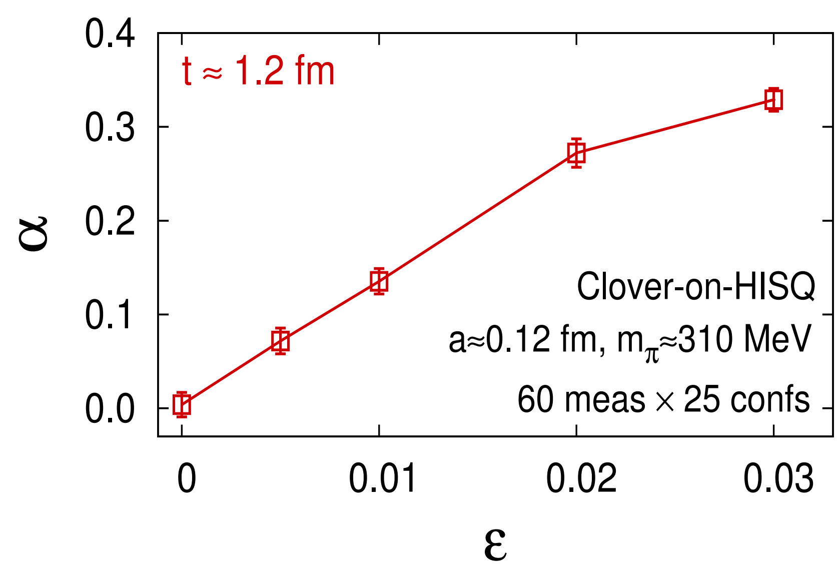


Does the method work?

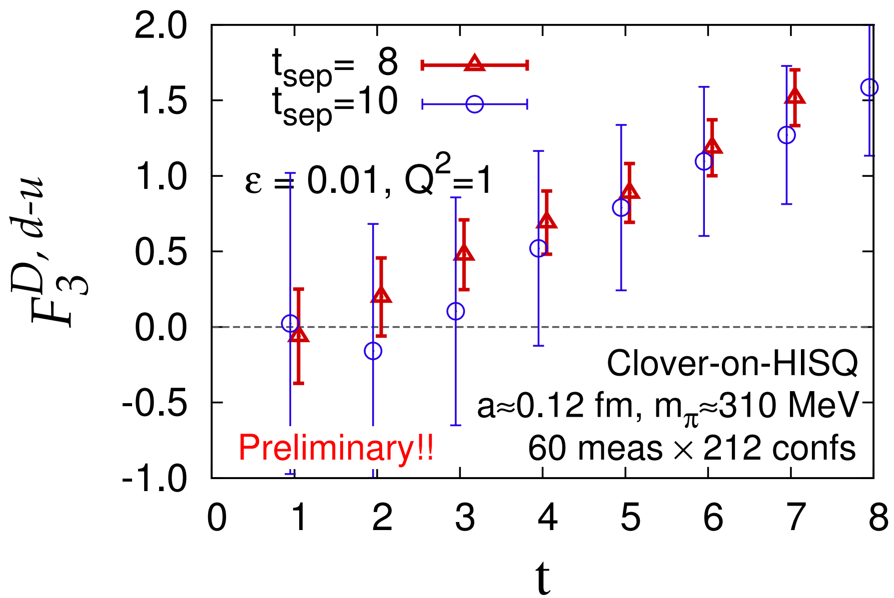
With CP violation $$u_N(p)_N\bar{u}(p) = e^{i\alpha_N\gamma_5}(i\not{p} + M_N)e^{i\alpha_N\gamma_5}$$

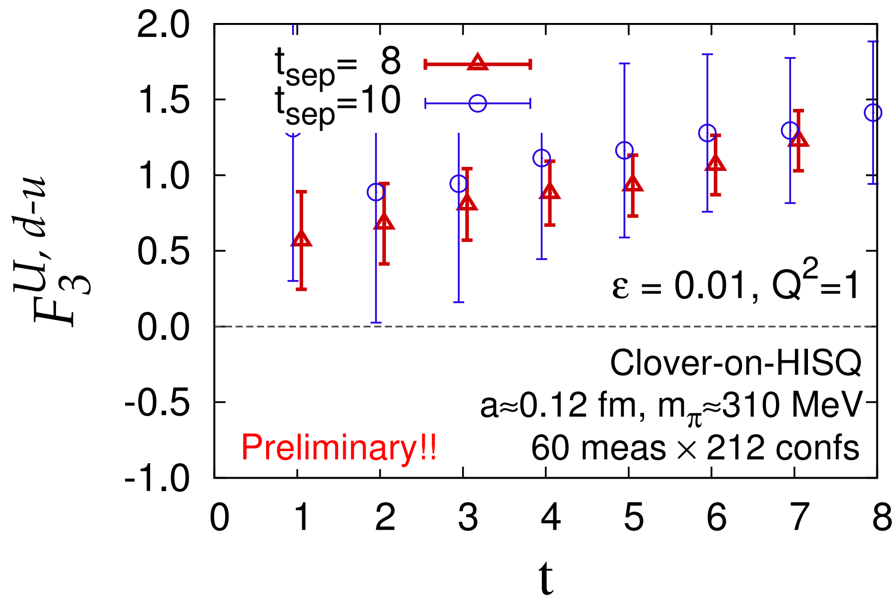
2-pt function: The phase α should be linear in ε for small ε





Connected part of F_3 with the chromo EDM operator





What Next

- Write up all the results obtained this year
- Develop and demonstrate signal (10% error) in all the pieces of the chromo EDM operator
- Resources for generating the lattices to do a credible study versus a, M_{π} and the lattice volume