

Spectral functions in Quark Gluon Plasma

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Spectral functions encode information on in-medium hadron properties, electromagnetic radiation and transport coefficients and therefore are crucial for understanding of the experimental results from RHIC in terms of QCD

1) What are the experimental signs of the transition to a new state of matter ?

⇒ Melting of hadron states

2) At what temperatures the matter created at RHIC is strongly or weakly coupled, can it flow ?

⇒ Shear viscosity and heavy quark diffusion constant

3) What is the temperature of the created matter ?

⇒ Thermal photons and dileptons, quarkonia

The perfect liquid created in RHIC

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a liquid.

Secretary of Energy Samuel Bodman

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

How small is the shear viscosity ?

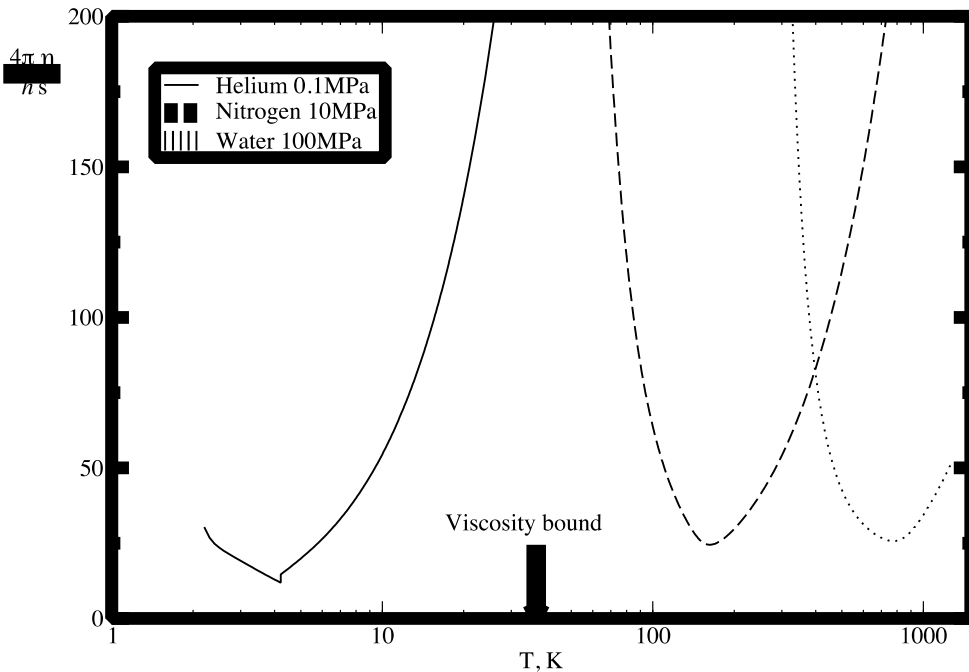
Validity of the hydrodynamics is governed by η/s

Hadron gas and QGP at very high temperature have large value η/s

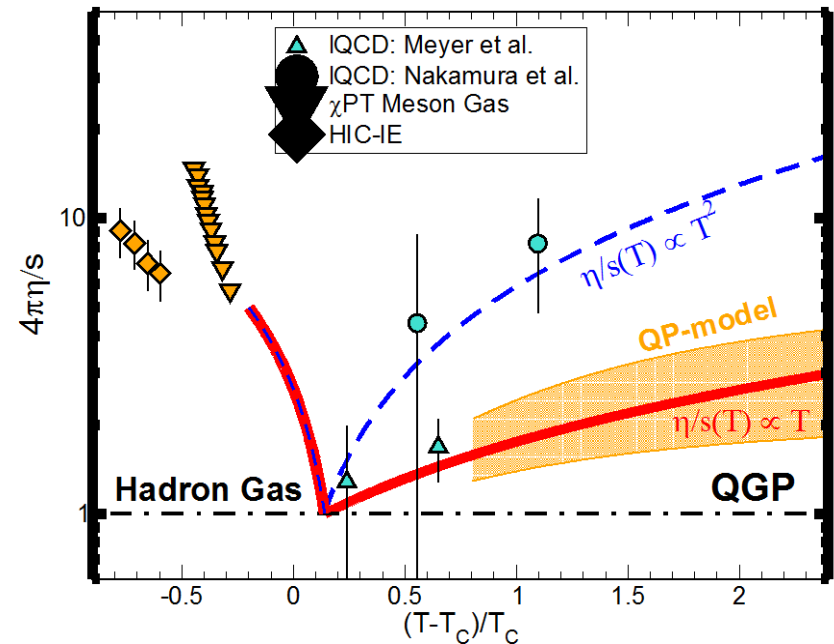
Super-symmetric gauge theories at strong coupling have small η/s with lower bound dictated by quantum mechanics $\eta/s > 1/(4\pi)$ (Kovtun, Son Starinets 2005)

\Rightarrow QGP near the transition temperature T_c has close to minimal η/s

Kovtun, Son Starinets, 2005



Csernai et al, 2013



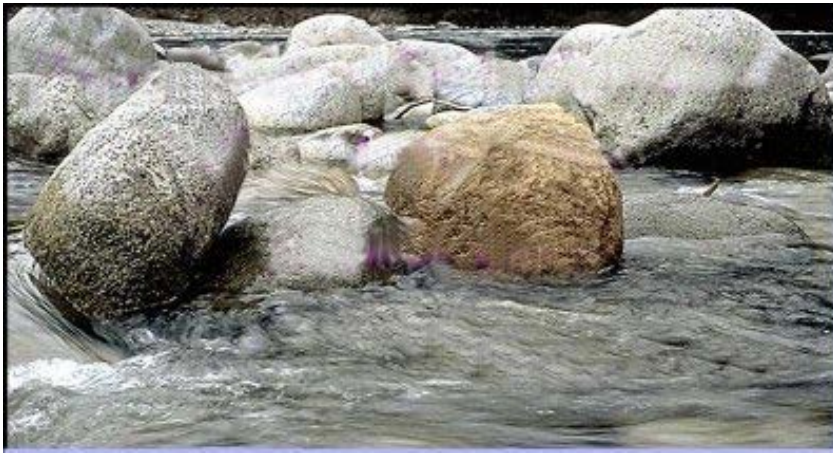
Extremely difficult to calculate in LQCD !

However, other transport coefficients are easier to calculate

Strongly coupled QGP and heavy quarks

Heavy quarks ($M_c \sim 1.5 \text{ GeV}$) flow in the strongly coupled QGP

Analogy from Jamie Nagle

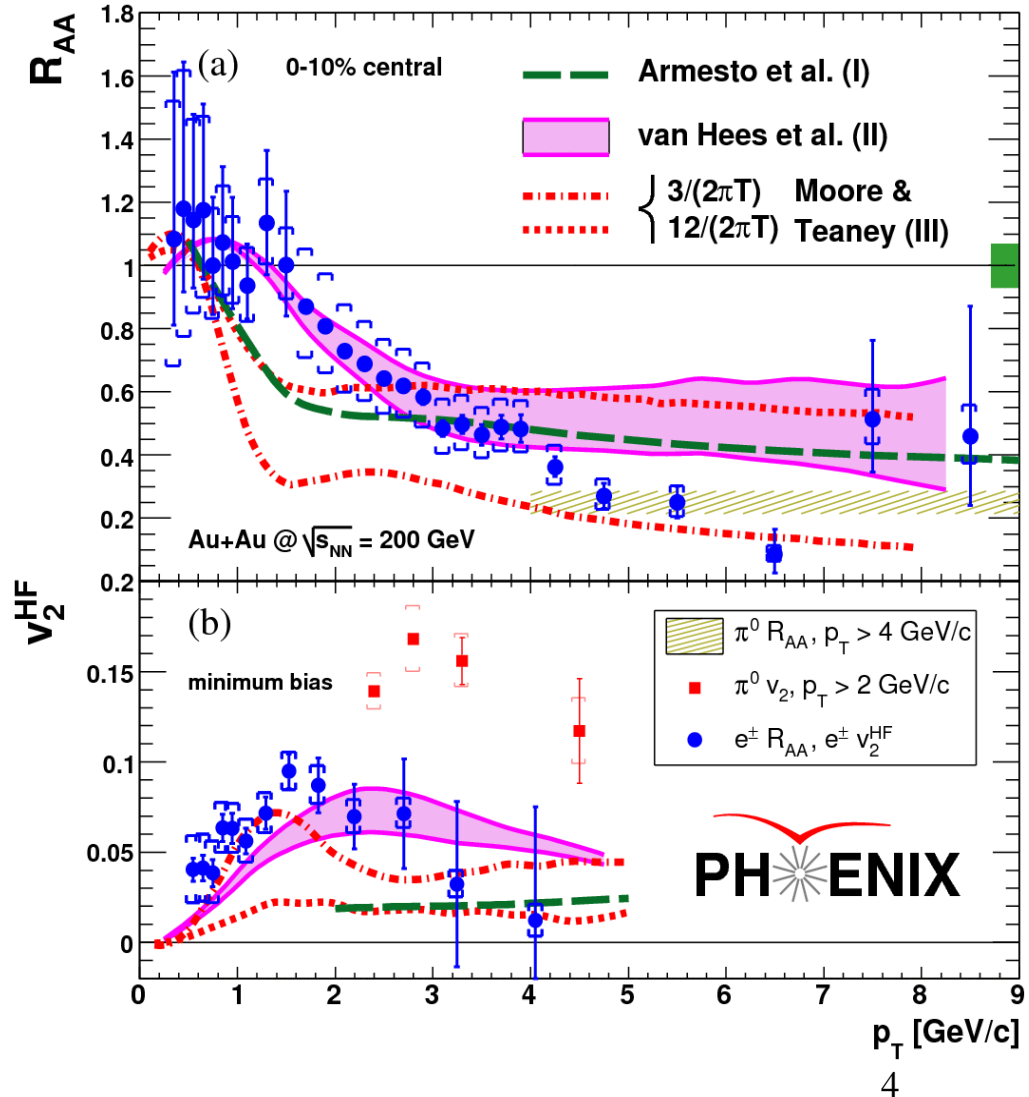


$t_{rel}^{heavy} \sim \frac{M_c}{T} t_{rel}^{light} \Rightarrow$ Langevin dynamics:

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \quad \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \quad D = \frac{T}{M\eta}$$

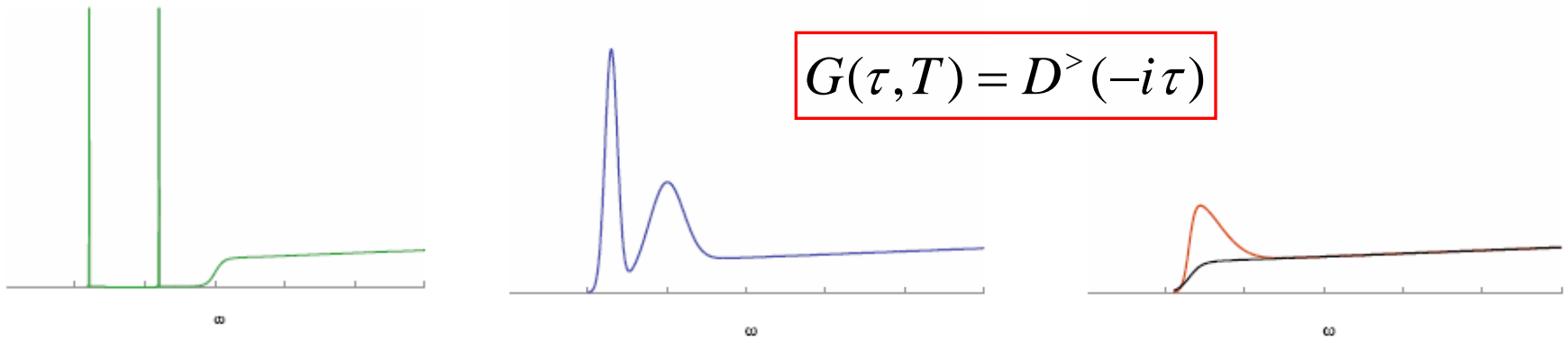


Spectral functions and lattice QCD (I)

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

$$\sigma(\omega, p, T) = \frac{1}{2\pi} \text{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [J(x, t), J(x, 0)] \rangle_T$$

Melting is seen as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$G(\tau, p, T) = \int d^3x e^{ipx} \langle J(x, -i\tau), J(x, 0) \rangle_T$$

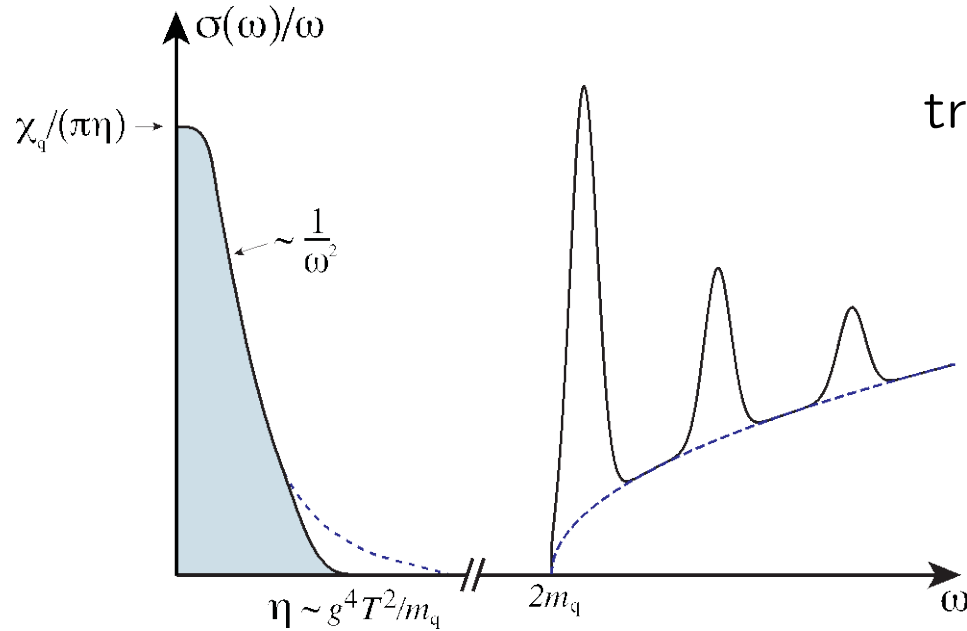
$$G(\tau, p, T) = \int_0^{\infty} d\omega \sigma(\omega, p, T) \frac{\cosh(\omega \cdot (\tau - \frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Need large temporal extent

Most calculations are done in Quenched QCD !

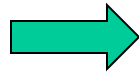
(no dynamical quark effects)

Spectral functions and lattice QCD (II)



Light vector meson spectral functions:

$$J_\mu = \bar{\psi} \gamma_\mu \psi$$



Thermal photons and dileptons provide information about the temperature of the medium produced in heavy ion collisions
 Low mass dileptons are sensitive probes of chiral symmetry restoration at $T > 0$

transport coefficients = $\lim_{\omega \rightarrow 0} \frac{\sigma(\omega)}{\omega}$

$$\sigma_{JJ}^{\text{low}}(\omega) = \chi \omega \frac{1}{\pi} \frac{T}{M} \frac{\eta}{\omega^2 + \eta^2}$$

P.P., Teaney, PRD 73 (06) 014508

thermal dilepton production rate
 (# of dileptons/photons per unit 4-volume)

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2} \frac{1}{e^{\omega/T} - 1} \frac{\sigma_{\mu\mu}(\omega, p, T)}{\omega^2 - p^2}$$

thermal photon production rate :

$$p \frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi} \frac{1}{e^{p/T} - 1} \sigma_{\mu\mu}(\omega = p, p, T)$$

2 massless quark (u and d) flavors are assumed; for arbitrary number of flavors $5/9 \rightarrow \sum_f Q_f^2$

Heavy quark diffusion constant from LQCD

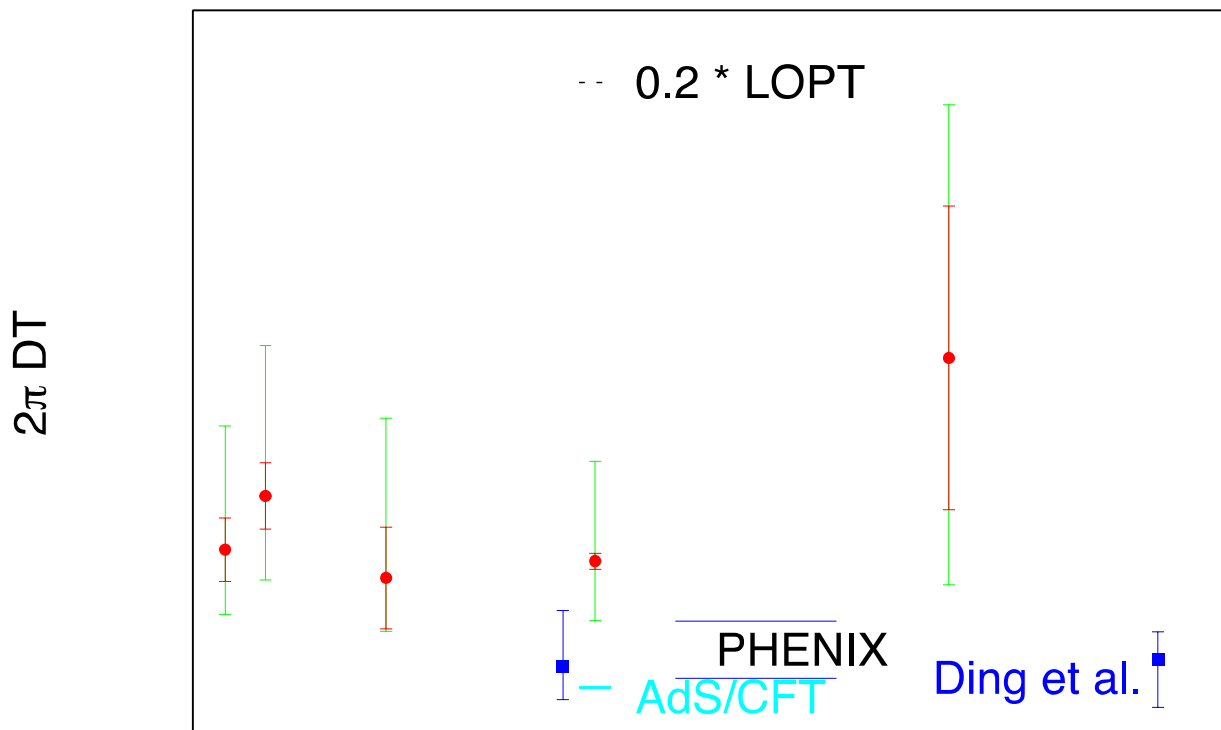
Direct method: determine the width of the transport peak,

Ding et al, arXiv:1204:4954, quenched $128 \times N_\tau$ lattices, $N_\tau=24-48$

Integrate out the heavy quark fields: $\langle J_i(\tau) J_i(0) \rangle \Rightarrow \langle E_i^a(\tau) E_i^a(0) \rangle$

Banarjee et al, arXiv:1109.5738, Kaczmarek et al, arXiv:1109:3941, $N_\tau=16-24$

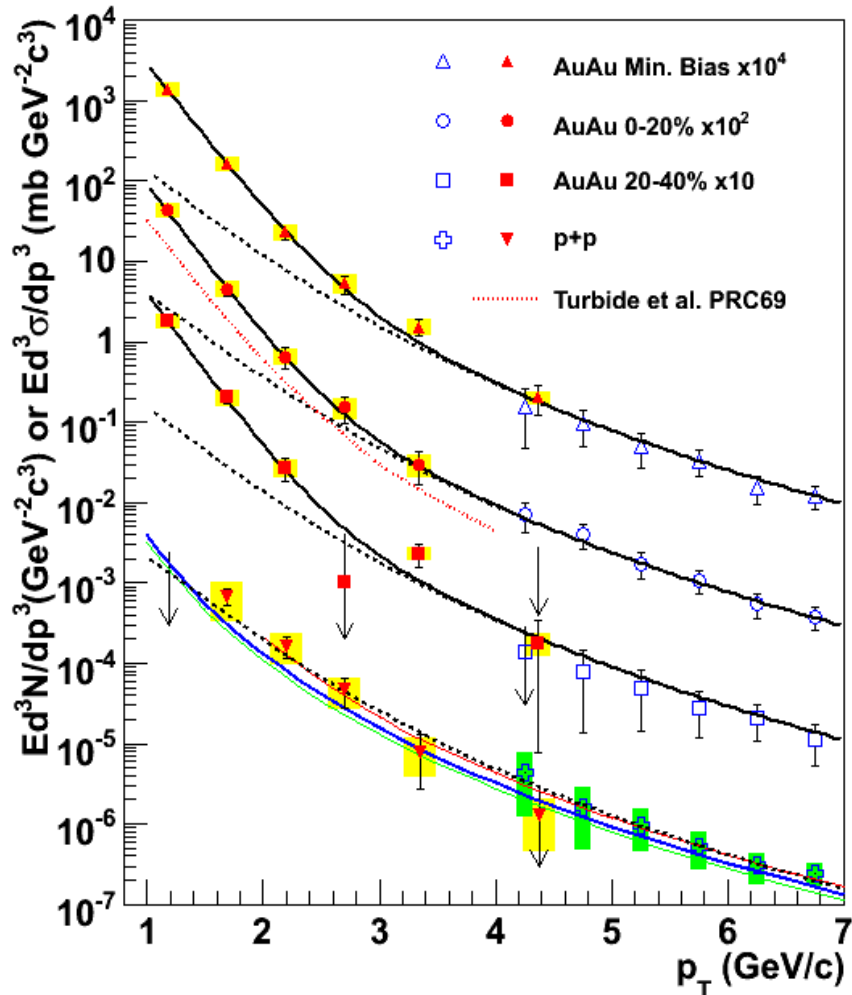
Lattice find values of D consistent with experiment and sQGP scenario



What is the temperature of the matter created in RHIC ?

Thermal photons: analog of black body radiation

$$T_{\text{ave}} = 221 \pm 19^{\text{stat}} \pm 19^{\text{syst}} \text{ MeV}$$



PRL104,132301(2010), arXiv:0804.4168

Perfect' Liquid Hot Enough to be Quark Soup Protons, neutrons melt to produce 'quark-gluon plasma' at RHIC

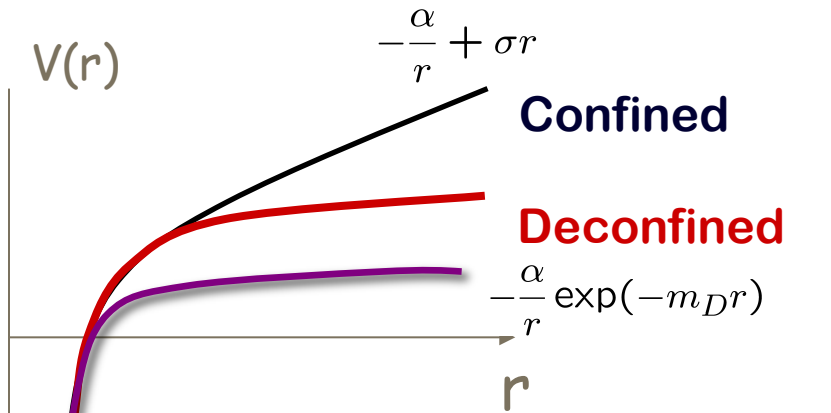
Monday, February 15, 2010

UPTON, NY — Recent analyses from the Relativistic Heavy Ion Collider (RHIC), a 2.4-mile-circumference “atom smasher” at the U.S. Department of Energy’s (DOE) Brookhaven National Laboratory, establish that collisions of gold ions traveling at nearly the speed of light have created matter at a temperature of about 4 trillion degrees Celsius — the hottest temperature ever reached in a laboratory, about 250,000 times hotter than the center of the Sun. This temperature, based upon measurements by the PHENIX collaboration at RHIC, is higher than the temperature needed to melt protons and neutrons into a plasma of quarks and gluons. Details of the findings will be published in *Physical Review Letters*.

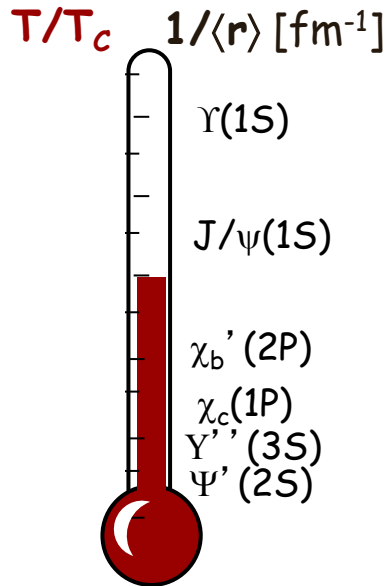
Guinness World Records, no longer encumbered by “book of,” recognized Brookhaven Lab for achieving the “Highest Man-Made Temperature.”

To estimate the temperature one needs to know
The photon rate as function of T :
very challenging task for LQCD

Dileptons and quarkonia



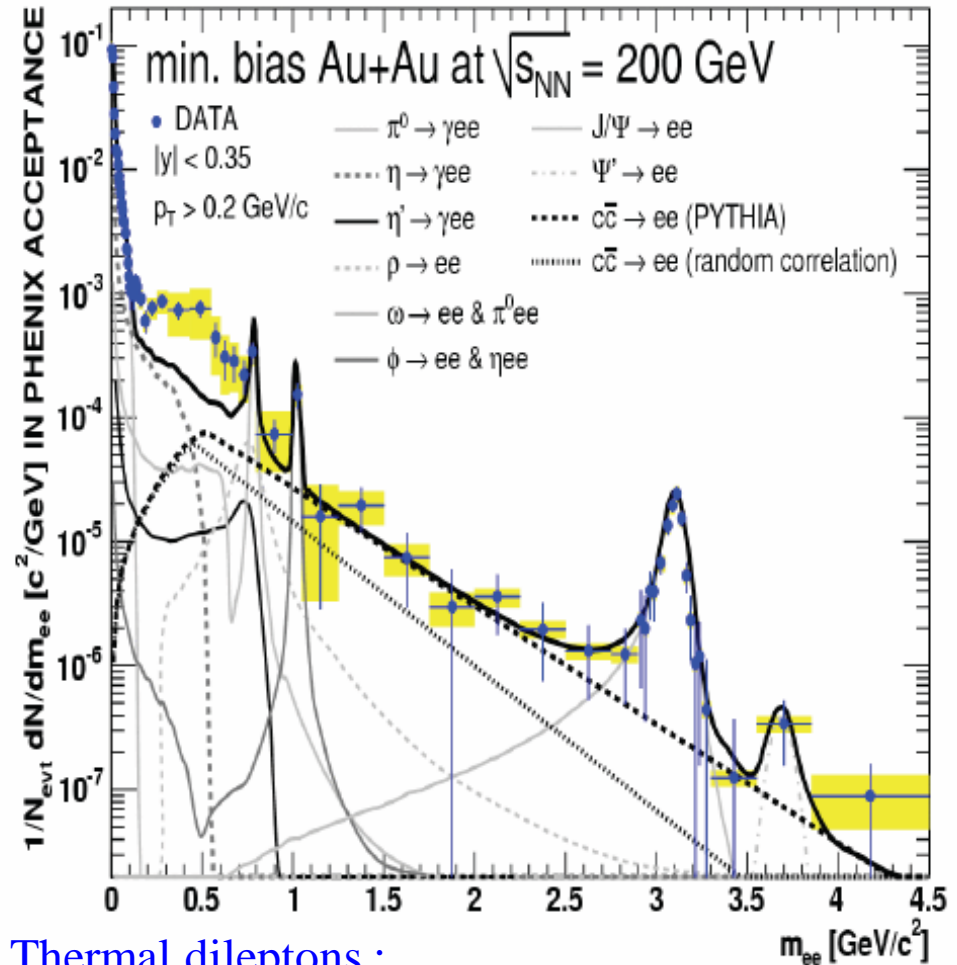
Matsui and Satz PLB 178 (86) 416



Melting depends on the binding energy



QGP thermometer



Thermal dileptons :

direct measurement of the temperature of the produced matter, melting of the rho meson, test consequences of chiral symmetry restoration

Quarkonium melting in RHIC and LQCD

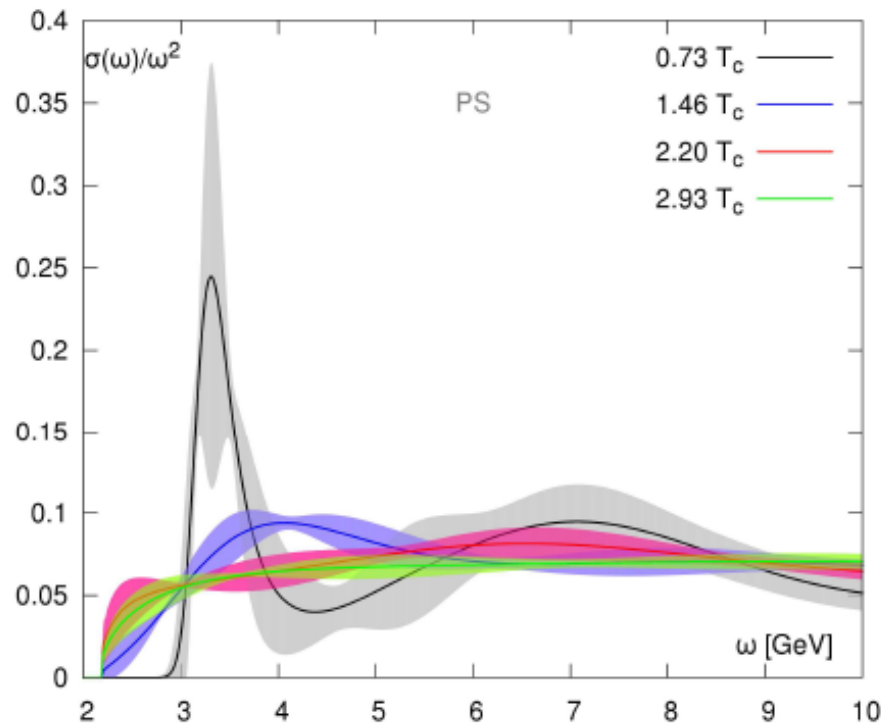
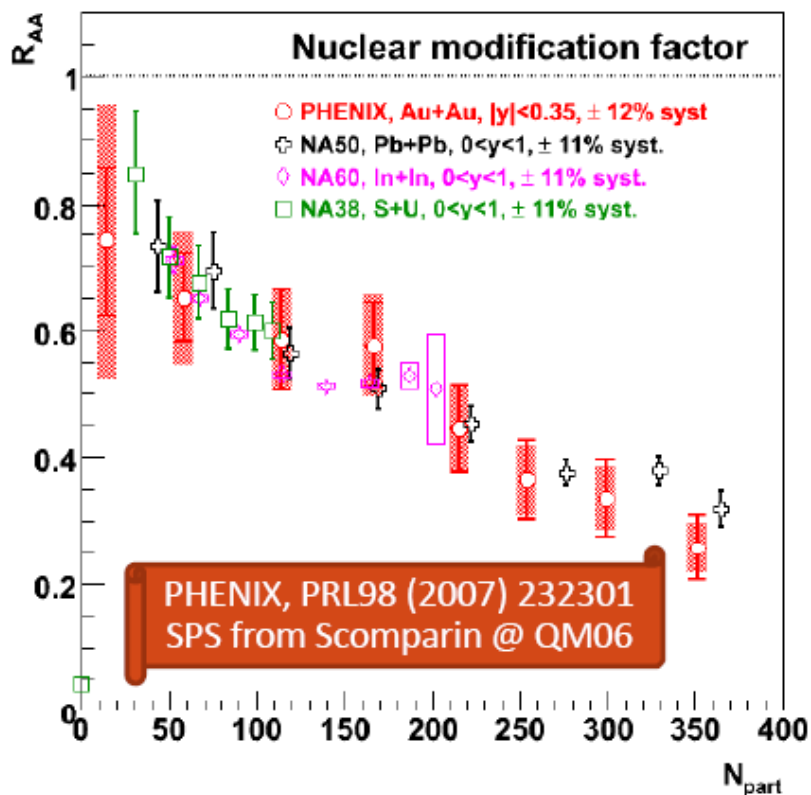
Charmonium yield is suppressed in heavy ion collisions => Melting of heavy quark bound states ?

Ding et al, arXiv:1011.0695 [hep-lat]

Charmonium at $T > 0$ in quenched LQCD

$$128^3 \times N_T, N_T = 96 - 24$$

$$a^{-1} = 18.97 \text{ GeV}$$



Quenched LQCD studies show that charmonium bound states dissolve at $1.5 T_c$

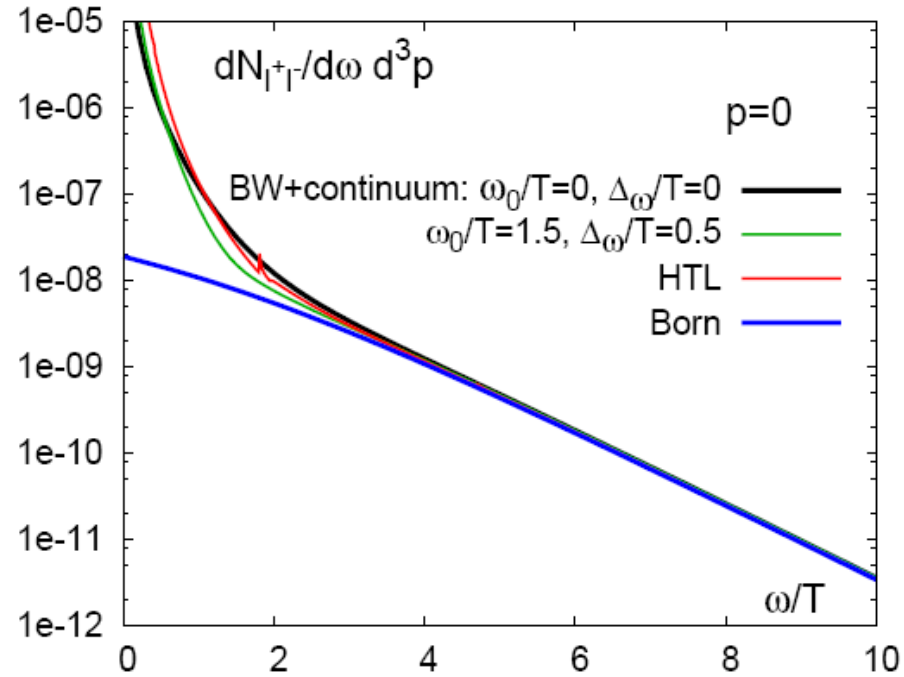
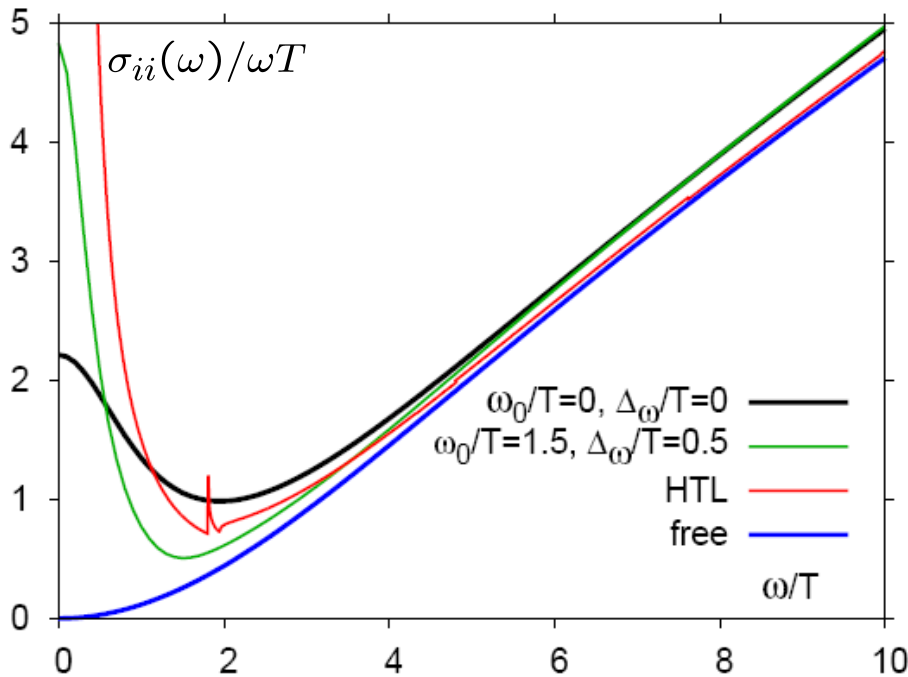
Similar conclusion in full QCD based on spatial meson correlators, Karsch et al, arXiv:1203.3770

However, a detailed study of the spectral functions is needed

Lattice calculations of the spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24, 32, 48$ ($a^{-1} = 9.4 - 18.8\text{GeV}$)

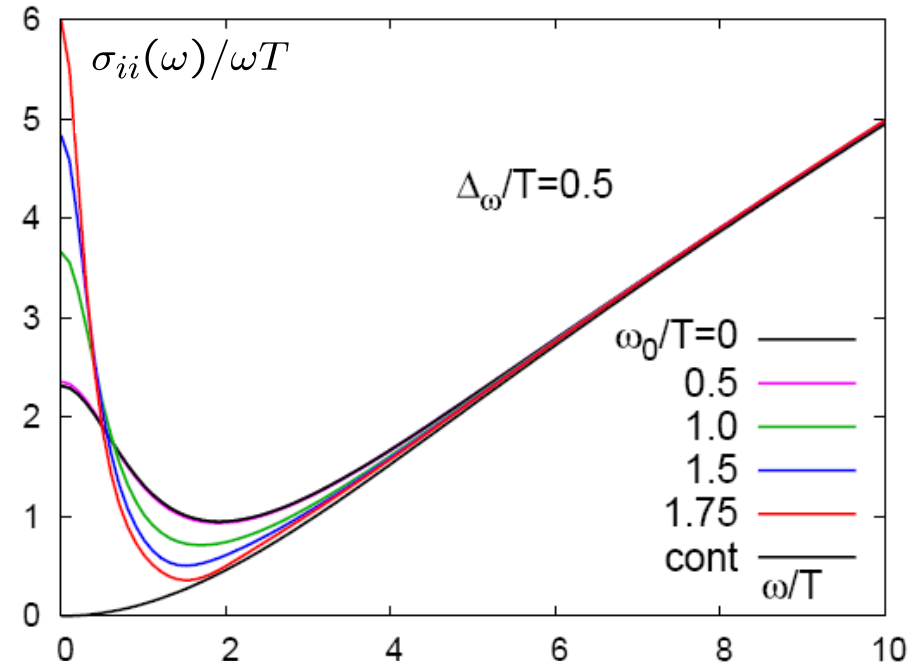
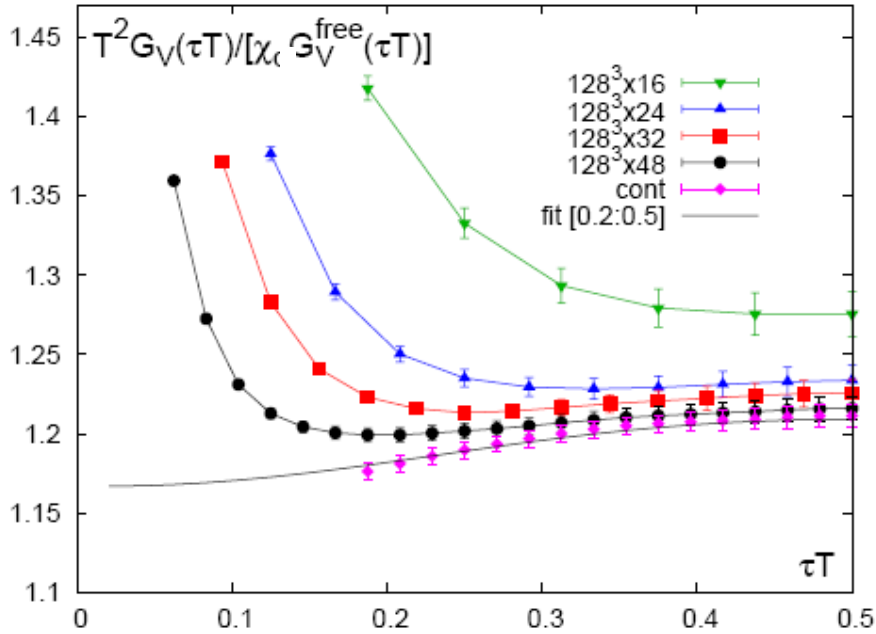


- The HTL resummed perturbative result diverges for $\omega \rightarrow 0$ limit
- The lattice results show significant enhancement over the LO (Born) result for small ω
- The lattice result is HTL result for $2 < \omega/T < 4$ but is much smaller for $\omega/T < 2$

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$$\sigma_{ii}(\omega) = \chi^{cBW} \frac{1}{\pi} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2} (1+k) \omega^2 \tanh(\omega/4T) \Theta(\omega_0, \Delta_\omega),$$

Fit parameters :
 c_{BW}, Γ, k

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega \Delta_\omega})^{-1}$$

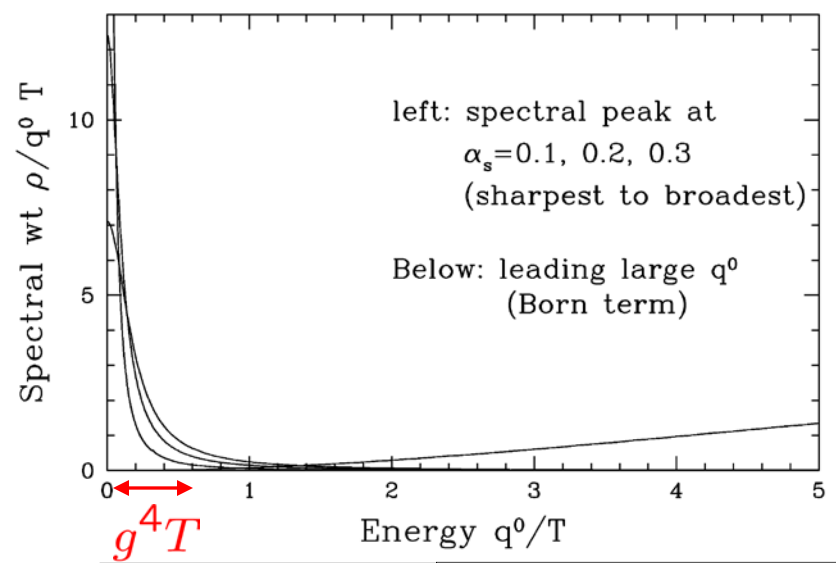
Different choices of :
 ω_0, Δ_ω

$$1/3 < \frac{1}{C_{em} T} \zeta < 1, \quad C_{em} = \sum_f Q_f^2$$

Strongly or weakly coupled QGP

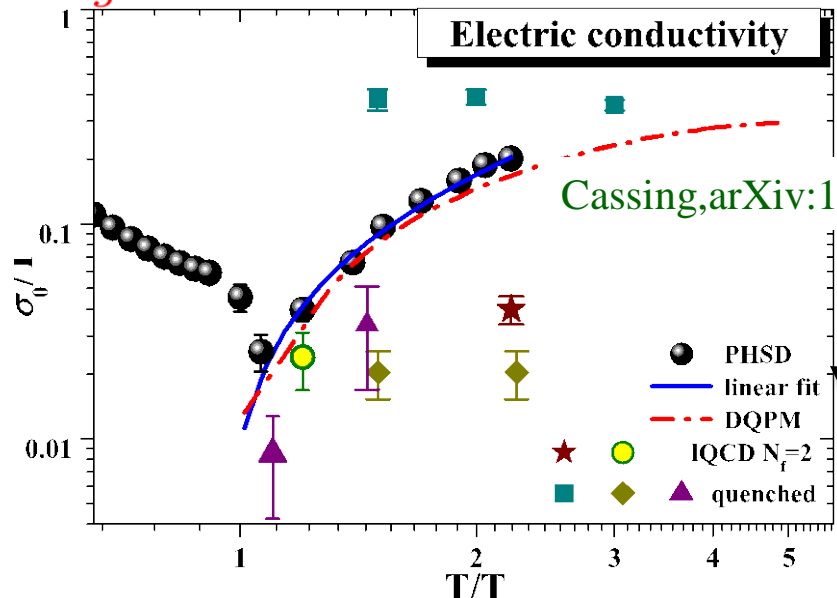
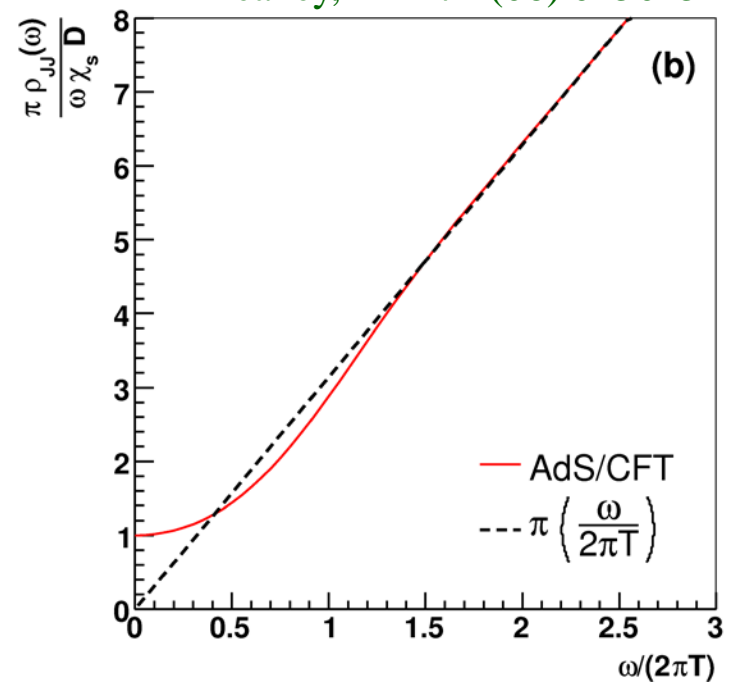
Weak coupling calculation of the EM current

spectral function in QCD [Moore, Robert, hep-ph/0607172](#)



vector current correlator in N=4 SUSY at strong coupling

[Teaney, PRD74 \(06\) 045025](#)



[Cassing, arXiv:1302.0906](#)

lattice results at $1.45T_c$ are closer to the weakly coupled QGP

LQCD results fit into the expected T -dependence

Next steps ...

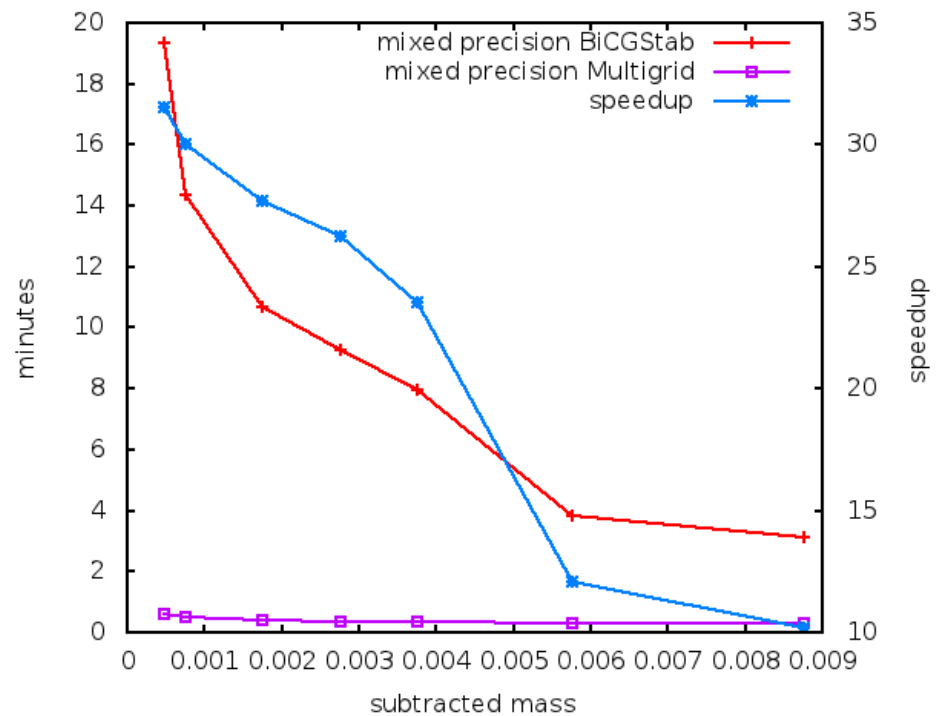
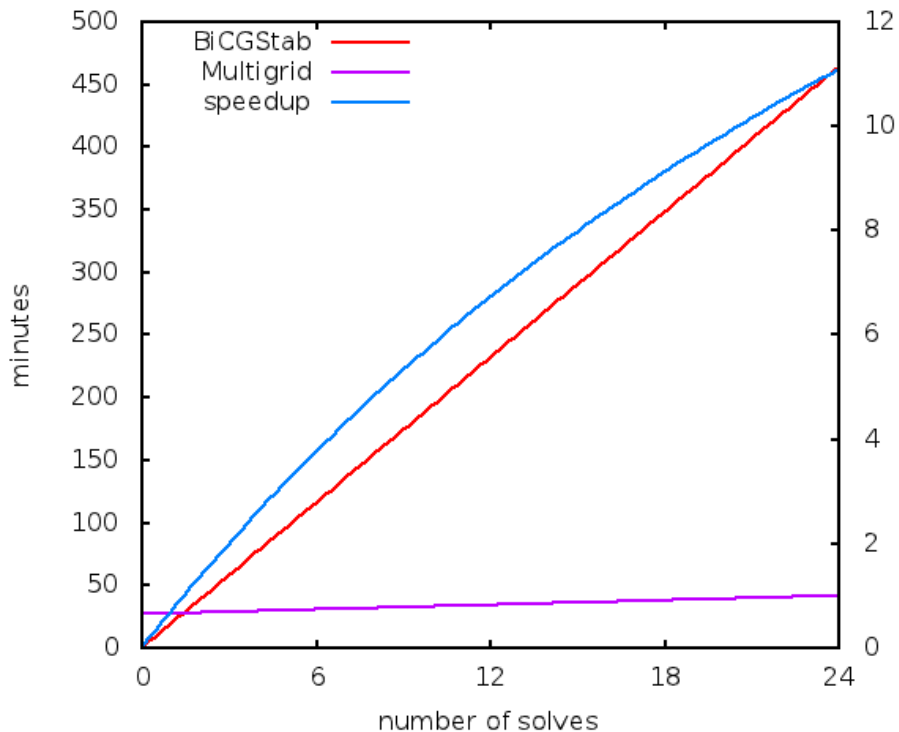
Extend quenched calculations to larger lattices, e.g. $196^3 \times 92$

Bottomonium spectral functions, non-zero momentum \Rightarrow photon rate

Significantly reduce the error on the correlation functions using multiple sources

\Rightarrow Multigrid algorithm (see talks by R. Brower)

Extend the calculations to full (unquenched) QCD, $128^3 \times 32$, $256^3 \times 64 \Rightarrow$ *multigrid algorithm?*



Osborne et al, PoS LATTICE 2010, 037 (2010)

Summary

Quenched lattice QCD calculation start to provide information on quarkonium melting, dilepton rate and two transport coefficients:

- a) Electric conductivity ζ
- b) Heavy quark diffusion constant D

Need to refine the quenched calculations and extend the calculations to full QCD