Hadron Structure from LQCD

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Importance:

- How are nucleons made up from the fundamental degrees of freedom (quarks and gluons)
- Understand the charge and magnetization distributions in the nucleon
- Complement key elements of DOE's experimental programs
  - quark distributions HERMES, Fermilab, LHC
  - form factors and GPDs: JLab
  - Contributions to the nucleon spin: JLab RHIC-spin, future EIC
  - Transverse momentum dependent distributions: JLab, RHIC-spin, Fermilab, future EIC
Relativistic Heavy Ion Collider

Proton structure

Monday, April 29, 13
Relativistic Heavy Ion Collider

Proton structure
Large Hadron Collider

Probe the fundamental forces in nature
<table>
<thead>
<tr>
<th>Year</th>
<th>#</th>
<th>Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>HP7</td>
<td>Measure the electromagnetic excitations of low-lying baryon states (&lt;2 GeV) and their transition form factors over the range $Q^2=0.1–7$ GeV$^2$ and measure the electro- and photo-production of final states with one and two pseudoscalar mesons.</td>
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<tr>
<td>2012</td>
<td>HP11</td>
<td>Measure the helicity-dependent and target-polarization-dependent cross-section differences for Deeply Virtual Compton Scattering (DVCS) off the proton and the neutron in order to extract accurate information on generalized parton distributions for parton momentum fractions, $x$, of 0.1 – 0.4, and squared momentum transfer, $t$, less than 0.5 GeV$^2$.</td>
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<tr>
<td>2013</td>
<td>HP8</td>
<td>Measure flavor-identified $q$ and $\bar{q}$ contributions to the spin of the proton via the longitudinal-spin asymmetry of $W$ production.</td>
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<tr>
<td>2013</td>
<td>HP12</td>
<td>Utilize polarized proton collisions at center of mass energies of 200 and 500 GeV, in combination with global QCD analyses, to determine if gluons have appreciable polarization over any range of momentum fraction between 1 and 30% of the momentum of a polarized proton.</td>
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<tr>
<td>2014</td>
<td>HP9</td>
<td>Perform lattice calculations in full QCD of nucleon form factors, low moments of nucleon structure functions and low moments of generalized parton distributions including flavor and spin dependence.</td>
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<tr>
<td>2014</td>
<td>HP10</td>
<td>Carry out ab initio microscopic studies of the structure and dynamics of light nuclei based on two-nucleon and many-nucleon forces and lattice QCD calculations of hadron interaction mechanisms relevant to the origin of the nucleon-nucleon interaction.</td>
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<tr>
<td>2015</td>
<td>HP13</td>
<td>Test unique QCD predictions for relations between single-transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering.</td>
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<tr>
<td>2018</td>
<td>HP14</td>
<td>Extract accurate information on spin-dependent and spin-averaged valence quark distributions to momentum fractions $x$ above 60% of the full nucleon momentum.</td>
</tr>
<tr>
<td>2018</td>
<td>HP15</td>
<td>The first results on the search for exotic mesons using photon beams will be completed.</td>
</tr>
</tbody>
</table>
We study strong interactions using the world's largest computers.
QCD

- The theory of strong interactions
- Coupling constant changes with scale
- Coupling constant becomes weak at high energies
- Characteristic scale arises $\Lambda_{\text{qcd}}$
- $\alpha_s(\Lambda_{\text{qcd}}) \sim 1$

Politzer, Gross, Wilczek 2004 Nobel prize
Low energy regime

- Interaction is strong
- Analytic calculations not under control
- Confinement
- Spontaneous chiral symmetry breaking
- Pions: Nambu–Goldstone bosons

Nambu 2008 Nobel prize
Particle masses

(Hadrons)

\[ m = A \Lambda_{qcd} \]

Can we calculate the dimensionless constants \( A \) ?

Quantum dynamics of strong interactions generates the mass of the visible matter
What does it take?

- Hadronic Scale: $1\text{fm} \sim 1 \times 10^{-13} \text{cm}$
- Lattice spacing $\ll 1\text{fm}$
  - take $a = 0.1\text{fm}$
- Lattice size $L_a >> 1\text{fm}$
  - take $L_a = 6\text{fm}$
- Lattice $64^4$
- Gauge degrees of freedom: $8 \times 4 \times 64^4 = 5.4 \times 10^8$
Monte Carlo Integration

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_{\mu, x} dU_\mu(x) \ \mathcal{O}[U, D(U)^{-1}] \ \det(D(U)^\dagger D(U))^{n_f/2} \ e^{-S_g(U)}
\]
Monte Carlo Integration

\[ \langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_{\mu,x} dU_\mu(x) \; \mathcal{O}[U, D(U)^{-1}] \; \text{det} \left( D(U) \dagger D(U) \right)^{n_f/2} \; e^{-S_g(U)} \]
Monte Carlo Integration

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Monte Carlo Integration

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int dU_{\mu}(x) \mathcal{O}[U, D(U)^{-1}] \det (D(U)^\dagger D(U))^{n_f/2} e^{-S_g(U)}
\]
Monte Carlo Integration

\[ \langle O \rangle = \frac{1}{Z} \int \prod_{\mu, x} dU_\mu(x) \; O[U, D(U)^{-1}] \; \text{det} \left( D(U)\dagger D(U) \right)^{n_f/2} \; e^{-S_g(U)} \]
Monte Carlo Integration

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_{\mu, x} dU_\mu(x) \mathcal{O}[U, D(U)^{-1}] \det (D(U) \dagger D(U))^{n_f/2} e^{-S_g(U)}
\]

Monte Carlo Evaluation

\[
\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)
\]

Statistical error

\[
\frac{1}{\sqrt{N}}
\]
Realistic Calculations

- Include the vacuum polarization effects
  - 2 light (up down) 1 heavy (strange)
  - ... and ... 1 very heavy (charm)

- Finite Volume
  - Compute in multiple and large volumes

- Continuum Limit
  - Compute with several lattice spacings

- Quark masses
  - Compute with several values for the quark masses
  - Study quark mass dependence of QCD
  - Physical light (up down) quark masses
Ken Wilson @ Lattice ‘89: “I still believe an extraordinary increase in computer power (10^8 is not enough) and equally powerful algorithmic advances will be necessary before a full interaction with experiment takes place”
Algorithm Improvements

- Hybrid Monte Carlo (HMC)
  - Stochastic sampling of the field configuration space
  - Improved representation of vacuum polarization effects
- Calculation of fermionic observables
  - Solve the Dirac equation
    \[ D(U)\chi = \phi \]
- Preconditioned solvers (eigCG), multi-grid, ...
Improved HMC

[Luscher 0710.5417]

- Uses an GCR with an AMG preconditioner
- [Luscher 0706.2298v4]
- Chronology reduces the refresh of the preconditioner
- Nearly removes critical slowing down
- Lattice $32^3 \times 64 \ a = 0.08\text{fm}$

\[N_{\text{ops}} \text{ [Tflops x year]} \]

- HMC '01
- DD-HMC '04
- deflated DD-HMC '07

physical point

plot by M. Luscher 1002.4232v2

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GPDs

X. Ji, D. Muller, A. Radyushkin (1994-1997)

Form Factors

Parton Distribution functions

Generalized Parton Distribution functions
Axial Charge

- Experimentally measured precisely $g_A = 1.2701 (25)$
- Lattice calculations underestimate it
- Recent work with better control of the systematics seems promising (QCDSF, LHPC, RBC/UKQCD)
Form Factors

Results from QCDSF, LHPC, RBC/UKQCD, ETM-QCD

Alexandrou et al arXiv:1303.5979
The nucleon sigma term

\[ \sigma_l \equiv m_l \langle N | \bar{u}u + \bar{d}d | N \rangle \quad \sigma_s \equiv m_s \langle N | \bar{s}s | N \rangle \]

- Dark Matter search experiments
- Controls uncertainty of hadronic uncertainties in dark matter cross sections for a large class of models
- LQCD determinations of the sigma term may prove important
Nucleon Sigma Term

\(0 \quad 40 \quad 80 \quad 120 \quad 160\)

\(\sigma_s (\text{MeV})\)

JLQCD 2008
JLQCD 2010
QCDSF 2011
Young & Thomas 2009
Toussaint & Freeman 2009
Martin–Camalich et al. 2010
Dürr et al. 2011
QCDSF–UKQCD 2011
Freeman & Toussaint 2012
Shanahan et al. 2012
JLQCD 2012
Ren et al. 2012
Engelhardt 2012

\(0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100\)

\(\sigma_f (\text{MeV})\)

Fukugita et al. 1995
Dong et al. 1996
SESAM 1998
Leinweber et al. 2000
Leinweber et al. 2003
Procura et al. 2003
Procura et al. 2006
ETM 2008
JLQCD 2008
QCDSF 2011
QCDSF 2012
Young & Thomas 2009
PACS–CS 2009
Martin–Camalich et al. 2010
Dürr et al. 2011
QCDSF–UKQCD 2011
Shanahan et al. 2012
Ren et al. 2012

Young et al. arXiv:1301.1765
Scalar and Tensor charges

- Search for new physics with precision measurements of Ultra Cold Neutrons decays
- The scalar and tensor iso-vector charges of the neutron are required to \(~\sim 10\%\)

Gupta et al. arXiv:1110.6448
Scalar and Tensor charges

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Gupta et al. arXiv:1110.6448
GPDs: Lattice QCD


Angular momentum

RBC/UKQCD:
gauge fields
2+1 flavors
domain wall fermions

MILC:
gauge fields
2+1 flavors
asqtad fermions

Puzzle:
Why the difference from experiment?

- Excited state contamination in nucleon structure
Moments of structure functions

Alexandrou et al. arXiv:1303.6818
The Proton Spin

• What is the contribution of the quark spin to the spin of the proton?

• EMC (1988) $Q^2 = 10 \text{ GeV}^2 \Delta \Sigma = 0.00(24)$

• SMC (1998) $Q^2 = 5 \text{ GeV}^2 \Delta \Sigma = 0.13(17)$

• Quarks contribute very little to the spin of the proton!

• $\Delta \Sigma$ should be 1 if all the proton spin is due to the spin of the quarks

• Spin crisis
The Proton Spin

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_z \]

or

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_z^q + J_z^g \]

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

• The quark angular momentum can be computed on the lattice

\[ J_z^q = \frac{1}{2} [A_{20}(0) + B_{20}(0)] \quad [\text{Ji,'98}] \]

• A and B are the generalized form factors of

\[ \mathcal{O}_{\mu \nu} = \bar{q} \gamma_{\{\mu} \hat{D}_{\nu\}} q = T_{\mu \nu} \]

[Mathur et. al. '00] [LHPC: Hagler et. al. '03] [QCDSF: Hagler et. al. '03]
• Quark orbital angular momentum almost zero
• Disconnected diagrams are missing

Proton SPIN

\[ m_\pi \text{ [GeV]} \]

\[ \mu = 2 \text{ GeV} \]

[J. Green et. al. arXiv:1111.0255]
Computational Needs

Gauge Field generation only

\[ \text{Cost}_{\text{traj}} = \left( \left( \frac{\text{fm}}{a} \right)^4 \left( \frac{L_s}{\text{fm}} \right)^3 \left( \frac{L_t}{\text{fm}} \right) \right)^{5/4} \cdot \left\{ B \cdot \left( \frac{\text{MeV}}{m_\pi} \right)^\gamma + A \right\} \text{ (core hours)} \]

Example:

Physical pion mass, \( a=0.075 \text{fm} \) 1K lattices, 6fm box

Costs about 120 TFlop-Years

.....
Conclusions

• Lattice QCD is a mature field
  • Direct comparison with experiment is now possible for several observables
  • Predictions are now emerging
• Hadron structure is now probed with lattice calculations
• LQCD calculations complement existing experimental program
• Improved precision is required
  • Control systematic errors
• Potential impact to searches for new physics
• multi-Petaflop computing is required for precision calculations