Lattice QCD
and
HEP Intensity Frontier

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RIKEN BNL Research Center

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RBC/UKQCD Collaborations

- BNL HEP Theory
  Chulwoo Jung, Taku Izubuchi, Taichi Kawanai, Hyung-Jin Kim, Christoph Lehner, Amarjit Soni
- RIKEN-BNL Research Center
  Tomomi Ishikawa, Taku Izubuchi
  Eigo Shintani, Shigemi Ohta
- Columbia University
  Norman Christ, Luchang Jin, Christopher Kelly Meifeng Lin (ALCF), Robert Mawhinney,
  Greg McGlynn, Jianglei Yu
  Hantao Yin, Daiqian Zhang
- University of Connecticut
  Tom Blum
  Michael Abramczyk
- Boston University
  Oliver Witzel
- ANL
  Meifeng Lin

14 current students,
~23 PhD theses since 2005

- + UKQCD Collaboration (2005-)
  - Univ. of Edinburgh
    1 faculty, 2 PostDocs, 2+1 students
  - Univ. of Southampton
    3 faculty, 2+1 Postdoc, 4 students
- + JLQCD (2012- , collaborating for physics measurement methods)
  - KEK, Tsukuba & Osaka Univ
- + LHP (2013-, collaboration for Nucleon structure) MIT, New Mexico, William&Mary,

Lattice QCD for Intensity Frontier

- Intensity experiment <-> [QCD corrections] ⊗ [Standard Model & Beyond]

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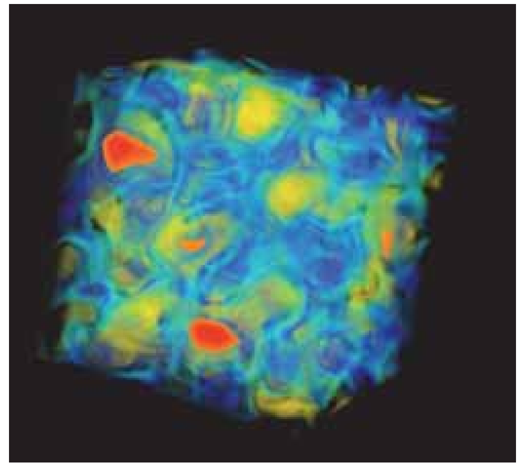
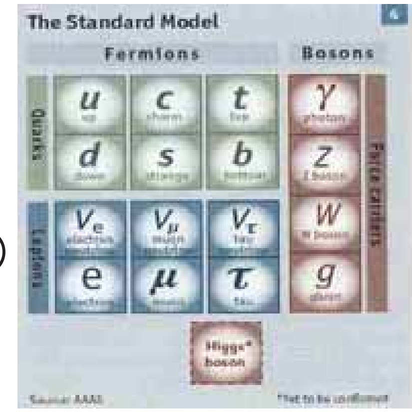


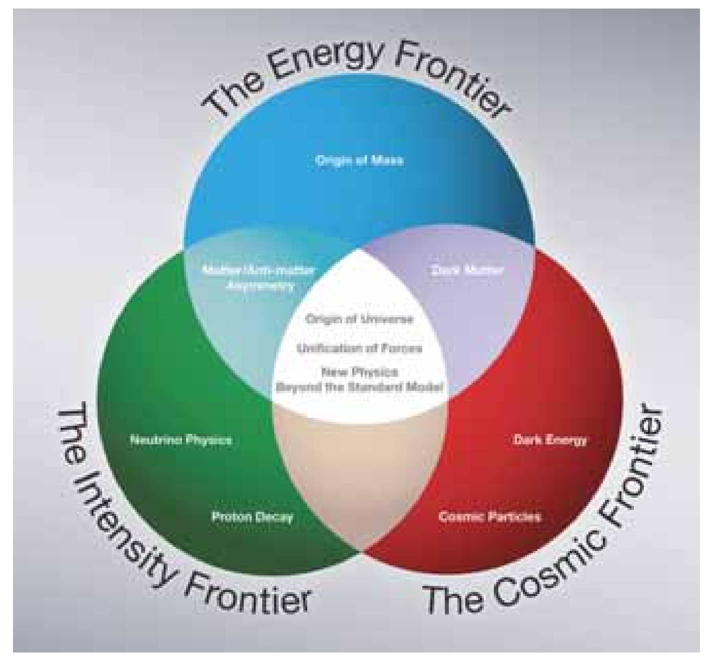
⊗



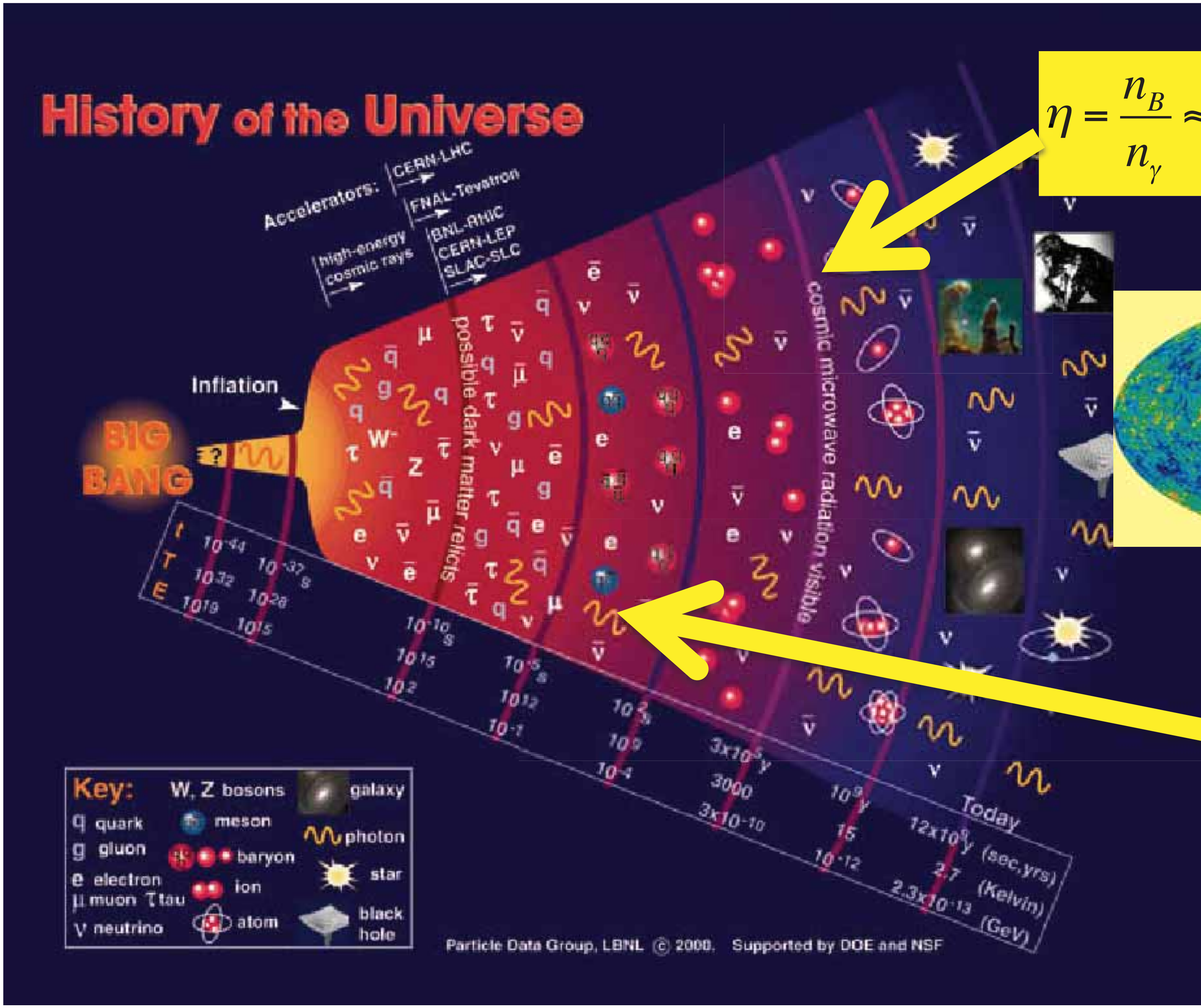
- Rare phenomenon / processes "Smoking Gun"
  - Proton/Neutron's Electric Dipole Moments (NEDM)
  - Proton decay

- Precision physics
  - Quark Flavor Physcs
    Cabibbo-Kobayashi-Maskawa (CKM) Theory
    [C. DeTar's talk]
    $K \to \pi\pi$ decay I=2 & I=0
  - Muon anomalous magnetici moment (g-2)μ

- All Mode Averaging (AMA) : statistical error reduction

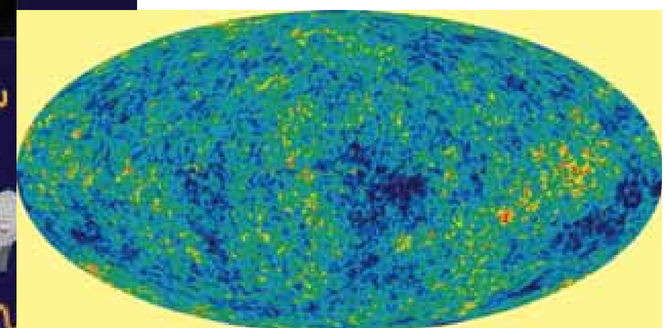


Matter dominant Universe



$$\eta = \frac{n_B}{n_\gamma} \approx (6.08 \pm 0.14) \times 10^{-10}$$

From CMB



$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \approx 10^{-18}$$

SM, Without net baryon number

Sakharov's three conditions

- In 1967, Sakharov pointed out three concurrent conditions for a small non-zero η in early universe (T ~ O(10) TeV)

1. Baryon number (B) violation
   To produce net Baryon from initial B=0
2. Violation of C (charge conjugation symmetry) and **CP** (parity and C)

   <B> != 0 , B : C, CP odd
3. departure from thermal equilibrium
   <B> = 0 in equilibrium from CPT

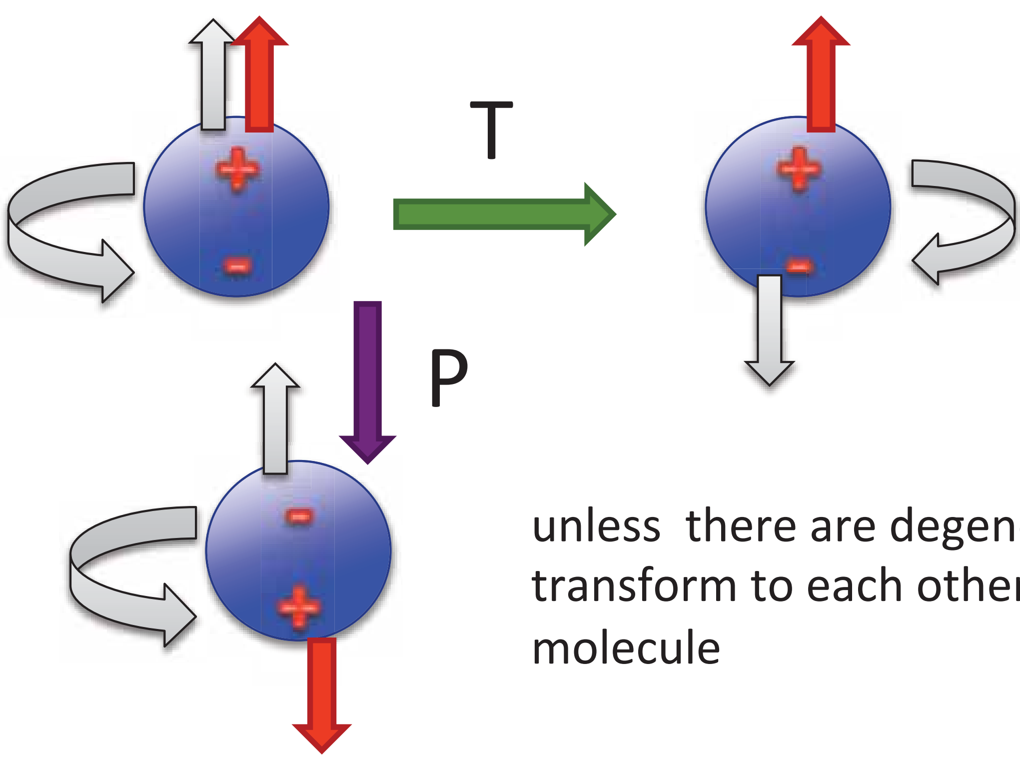
P & CP violation and Electric Dipole Moments (EDM)

- Electric Dipole Moment d

  energy shifts in an electric field E

$$\Delta H = \vec{d} \bullet \vec{E}$$

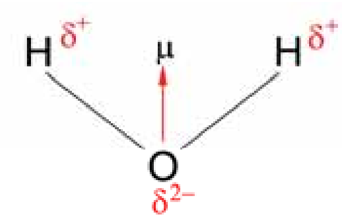
- A nonzero EDM is a signature of P and T (CP through CPT) violation



exp: $\Delta H \sim 10^{-6}$ Hz $\sim 10^{-21}$ eV
$\rightarrow$ $|d| < \Delta H/E \sim 10^{-25}$ e cm

if theo: $d \sim 10^{-2} \times 1$ MeV / Λ^2_{CP}
$\rightarrow$ Λ_{CP} >~ O(1) TeV

unless there are degenerate ground states transform to each other by Parity c.f. Water molecule



CP symmetry breaking from QCD

- QCD
  - θ term in the QCD Lagrangian:

$$\mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M$$

renormalizable and CP-violation comes due to topological charge density.

  - EDM experiment provides very strong constraint on
    $\Rightarrow$ θ and $\arg\det M$ need to be unnaturally canceled !
    strong CP problem, unless massless quark(s)

$$|d_N^{\mathrm{exp}}| < 2.9 \times 10^{-26}\,\mathrm{e}\cdot\mathrm{cm} \qquad \bar{\theta} < 10^{-9}$$

  - up quark mass less ?
  - Axion ?
  - contribution from CKM is very small < 1e-30 e cm

Constraint on nEDM

- The present and future experiments are aiming to check/exclude of MSSM

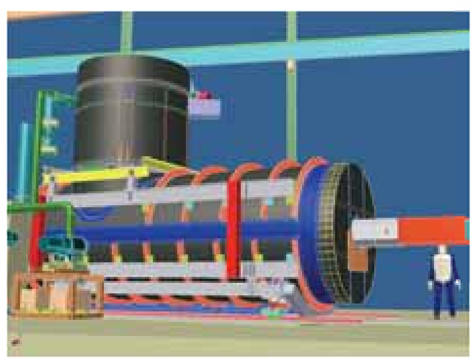
Harris, 0709.3100

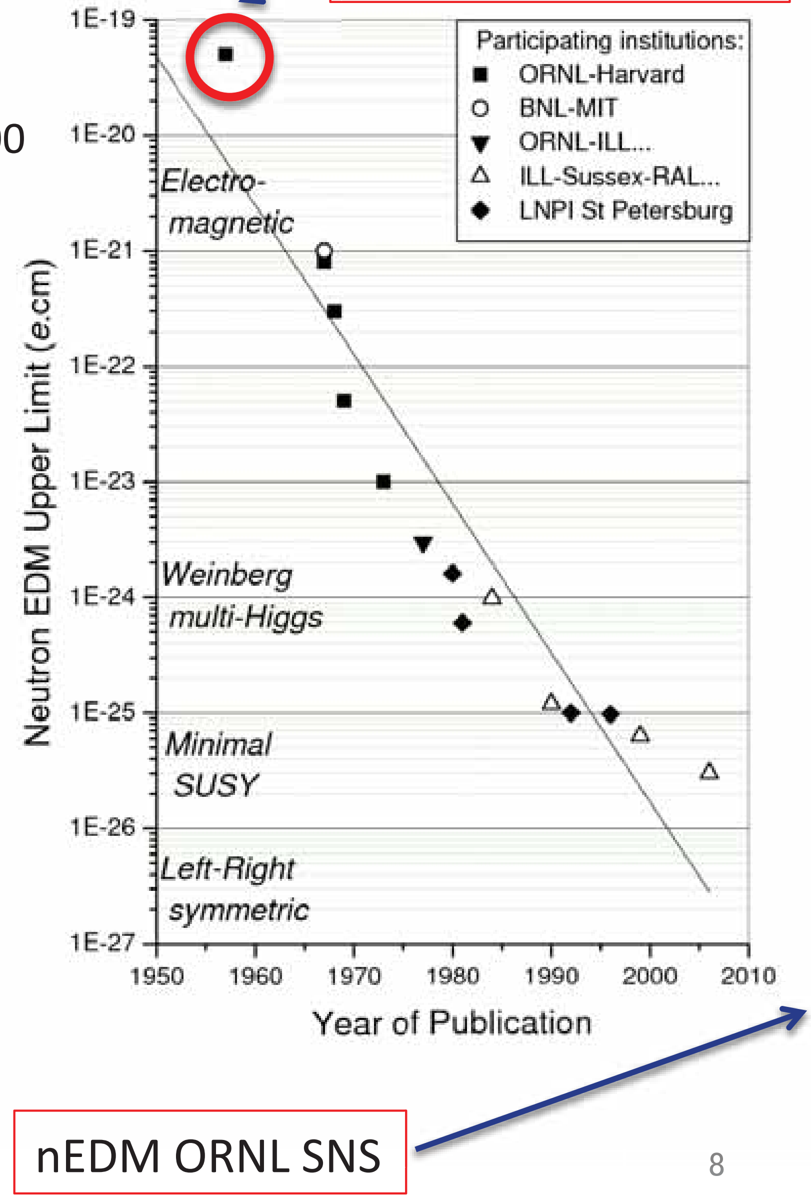
pEDM @ BNL
nEDM @ ORNL, PSI, ILL, J-PARC, TRIUMF ,FNAL, FRM2, …
charged hadrons @ COSY

⇒ a sensitivity of 10^{-29} e·cm !

- Current theoretical estimations are based on quark model, sum rules, …

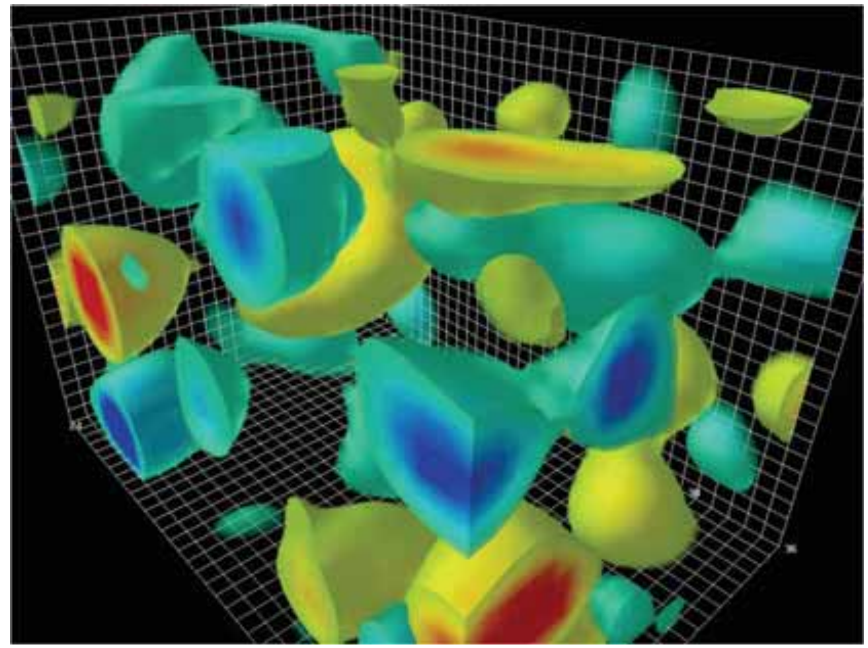
non-perturbative computations of EDM $d_n(\theta, d_q, d_q^c, \ldots)$ are necessary





EDM from lattice QCD
[E. Shintani, T.Blum]

- Three ingredients
  - QCD vacuum samples
  - source of CP violation
    - *Reweighting from CP symmetric vacuum*
    - *Dynamical simulation*
  - Polarized Nucleons



[D. Leinweber]

- Two methods
  - Measure a spin splitting of energy
  - Form Factor

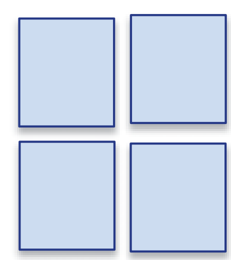
CP violation on lattice : Reweighting

- Source of CP violation (Θ in our case)

$$S_\theta = i\frac{\theta}{32\pi^2}\int d^4x\ \mathrm{tr}[\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}G^{\mu\nu}]$$

$$= i\theta\ Q_{\mathrm{top}}$$

- Topological charge is measured either by gluonic observable GG^* or by counting zero mode of chiral fermions

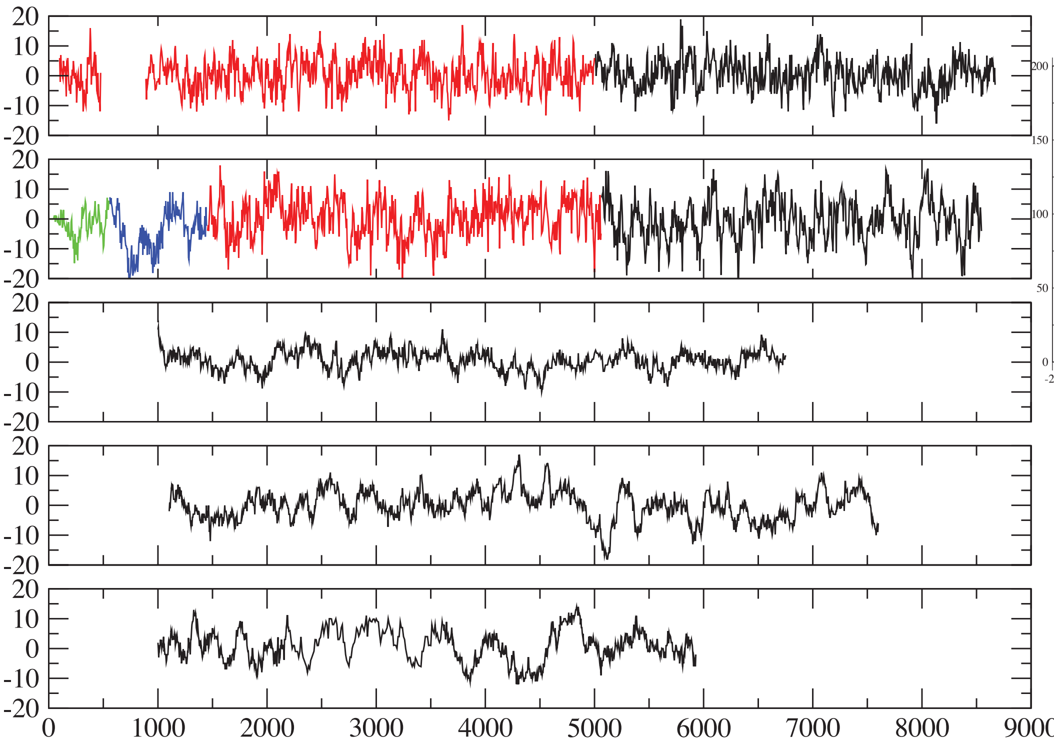
$Q \rightarrow \Sigma\ G_{12}\, G_{34}$, $G_{\mu\nu} = \mathrm{Im}$ 

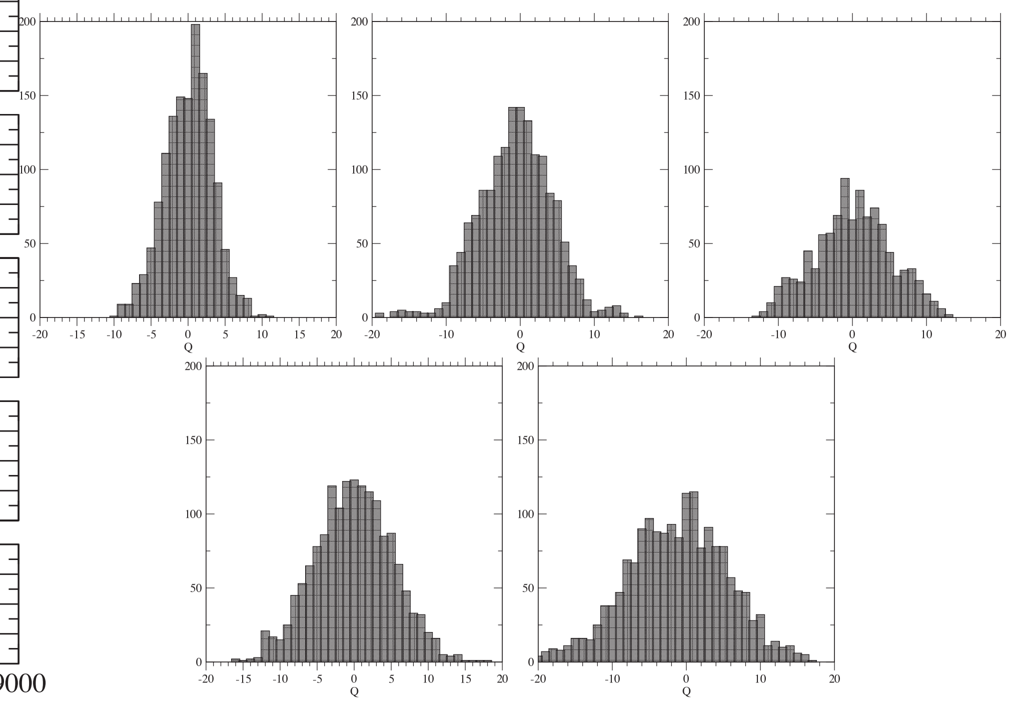
- Θ =0 lattice QCD ensemble is generated, then each sample of QCD vacuum are reweighted using topological charge

$$\langle \mathcal{O} \rangle_\theta = \langle \mathcal{O} e^{i\theta Q} \rangle_{\theta=0}$$

Qtop on lattice (Θ=0)

- Qtop history in simulation Nf=2+1 DWF, [RBC/UKQCD]
- 1/a= 1.73, 2.28 GeV
- mps =290 – 420 MeV





Mpi=330, 420 MeV

Chiral symmetry & EDM

$$q(x) \rightarrow e^{i\gamma_5\theta} q(x)$$
$$\bar{q}(x) \rightarrow \bar{q}(x) e^{-i\gamma_5\theta}$$

- Chiral symmetry is broken by lattice systematic error

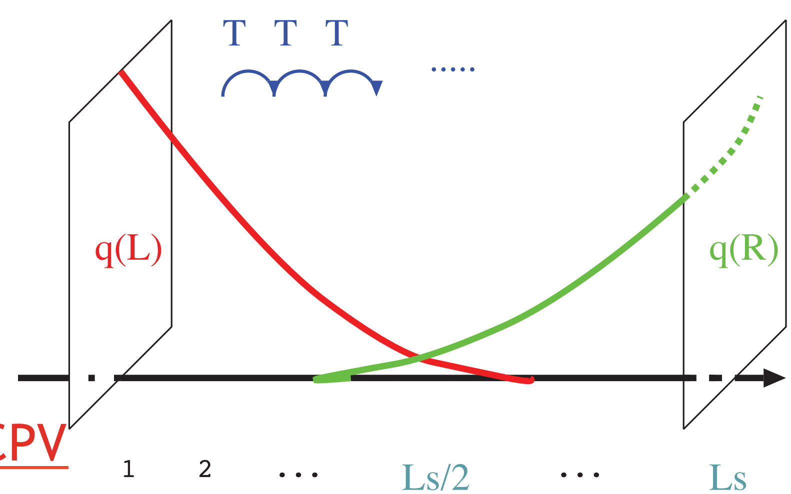
  for Wilson-type quarks, which has "wrong" Pauli term by O(a)

$$\mathcal{L}_{\rm Wilson} = \mathcal{L}_{\rm QCD} + ca\bar{q}\sigma_{\mu\nu}\cdot F_{\mu\nu}q$$

- CP violation from θ or other BSM operators introduce extra artificial CP violation in simulation.

- In fact, chiral rotation of valence quark is unobservable in continuum theory, and the EDM signal measured in Wilson quark due to valence quark's θ is unphysical, which should be carefully removed by taking continuum limit a → 0 [S. Aoki-Gockschu, Manohar, Sharpe et al. Phys.Rev.Lett. 65 (1990) 1092-1095 (1990)]

→ Our choice : chiral lattice quark called domain-wall fermions (DWF)
[97 Blum Soni, 99 CP-PACS, 00- RBC, 05 RBC/UKQCD…]



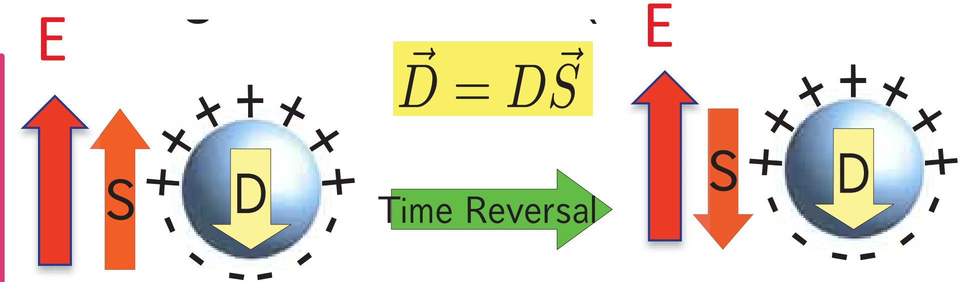
R. Gupta et al. Clover fermion for BSM CPV

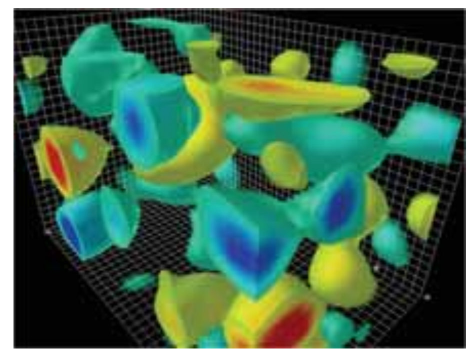
EDM Computations on Lattice

- Measure energies with external Electric field

$$\frac{\langle N_\uparrow(t)\bar{N}_\uparrow(t_0)\rangle}{\langle N_\downarrow(t)\bar{N}_\downarrow(t_0)\rangle} \to Ce^{\Delta M t}$$

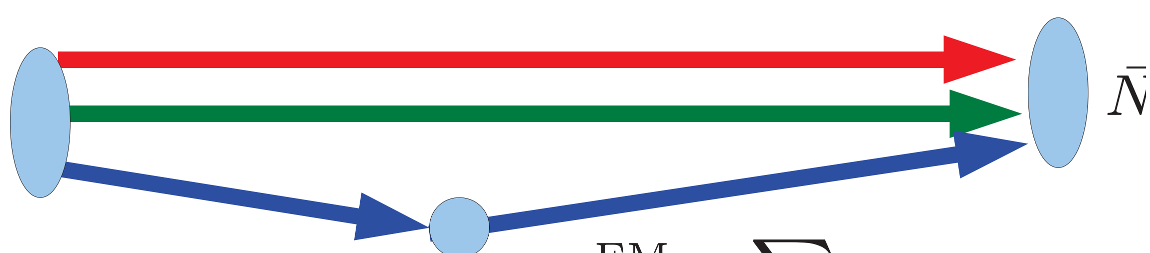
$$\Delta M = M_N(E,\uparrow) - M_N(E,\downarrow) = -2D_N(\theta) S\cdot E$$





- Form factors

$$\langle N(p')|V_\mu^{\mathrm{EM}}(q)|N(p)\rangle = F_1(q^2)\gamma_\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} + F_3(q^2)\frac{\sigma^{\mu\nu}q^\nu\gamma^5}{2m_N}$$



$$V_\mu^{\mathrm{EM}} = \sum_q e_q \bar{q}\gamma_\mu q$$

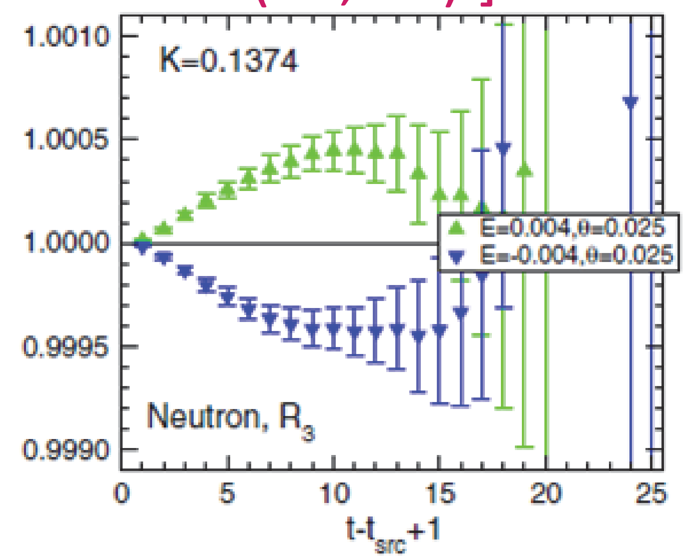
$$d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$$

External Electric field method

[E.Shintani et al. (06, 07)]

- Ratio of spin up and down

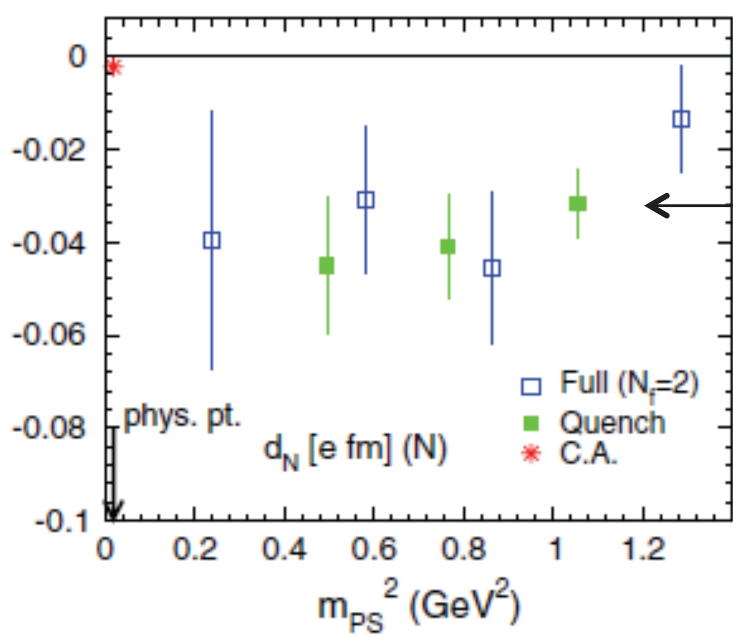
$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\rm up}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\rm down}_{\theta,E}} \simeq 1 + d_N E\theta t$$



Linear response, gradient is a signal of EDM.

- Reweighting works well for small real θ
- Temporal periodicity is broken by electric field.

  ⇒ additional systematic effects



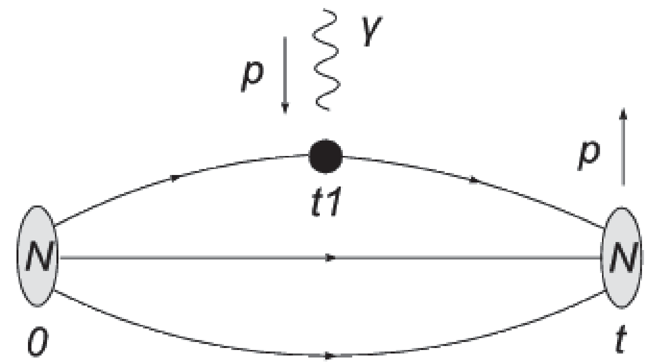
Full QCD with clover fermion:

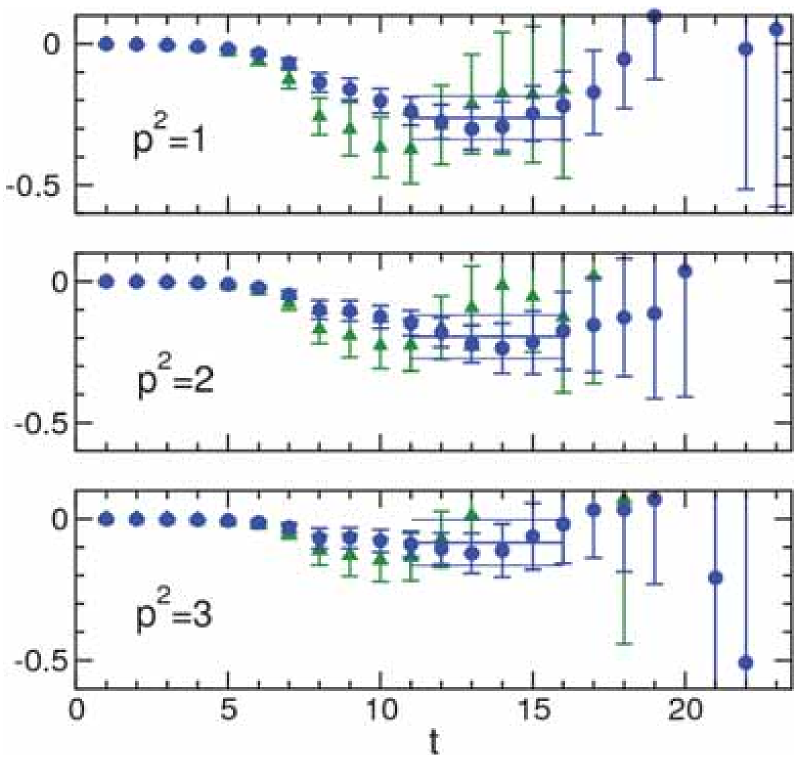
- There seems to be no significant difference between quench and full QCD for this heavy quark mass (pion mass >~ 500 MeV) .
- Statistical error is still large.
- Finite size effect from breaking of temporal periodicity is also significant

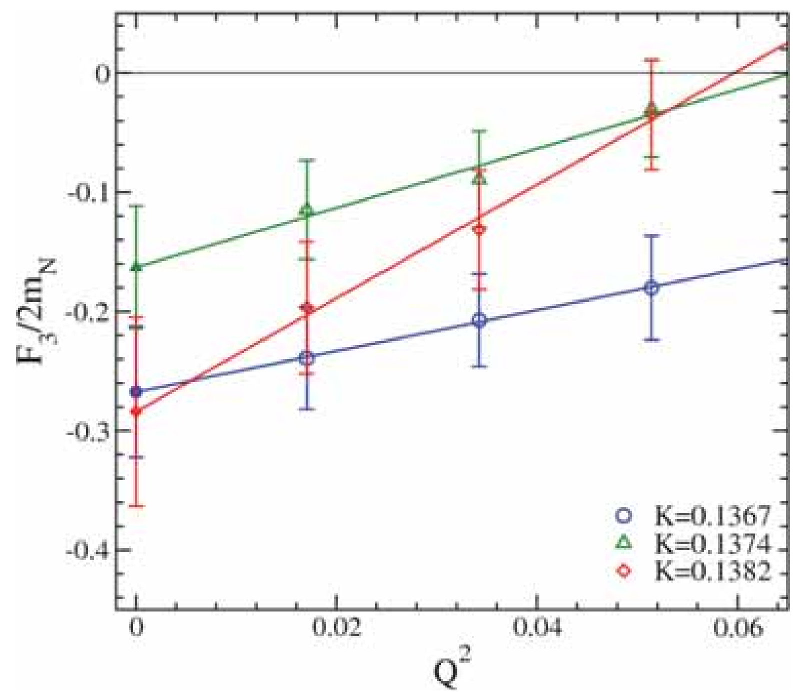
Form factor $F_3(q^2)$

F. Berruto et al. (05)
E.Shintani et al. (05, 08)

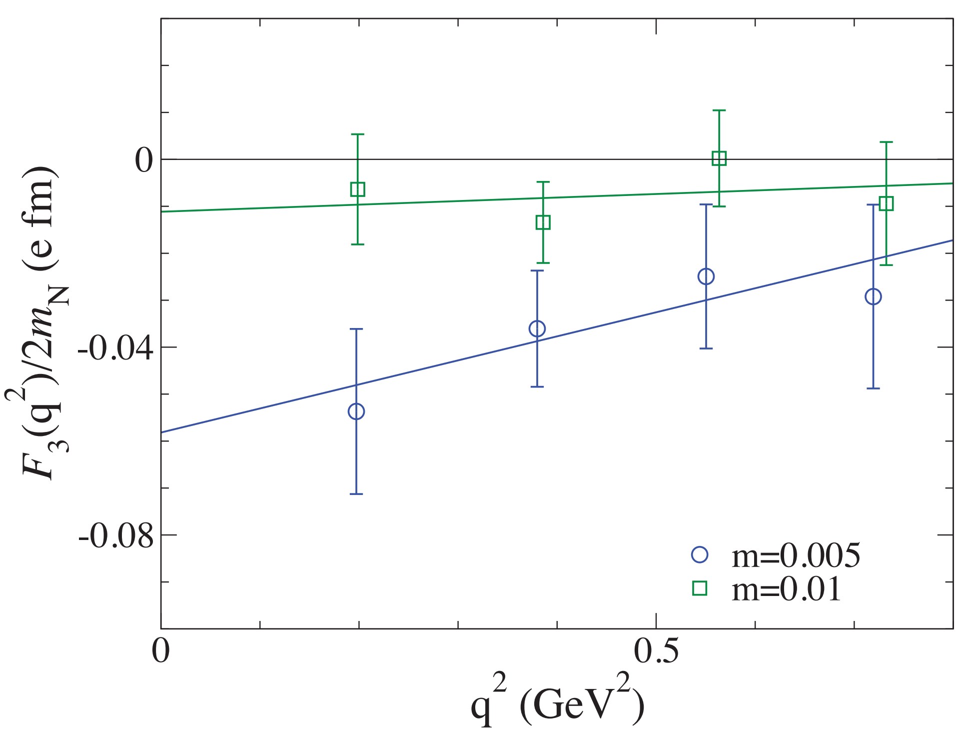
- Matrix element in θ vacuum
  - N_f=2 clover fermion
  - Size is $24^3 \times 48$ lattice (~2 fm^3)
  - Signal appears in 11 -- 16
  - $Q^2 \to 0$ limit with linear func.

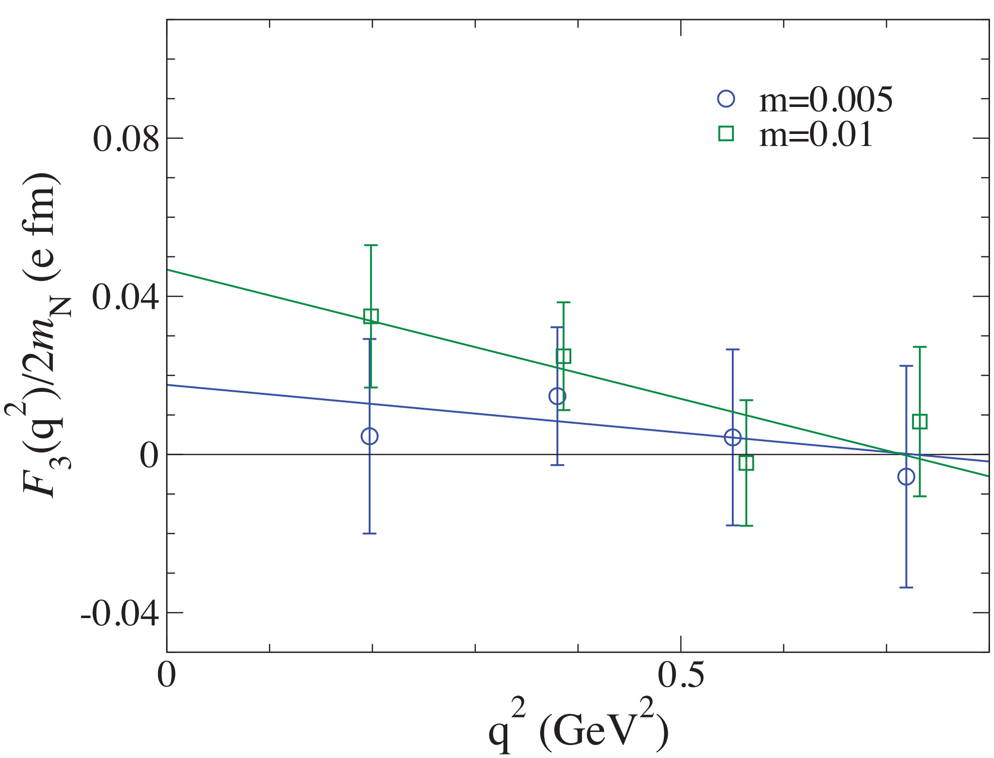






$F_3(q^2)$ vs q^2





- Mpi=330 MeV (mf=0.005) 600 config x 32 AMA = 19.2 K measurements
- Mpi=420 MeV (mf=0.01) 400 config x 32 AMA = 12.8 K measurements
- (iso-vector) CP violating g(π-N-N) is related to the slope of F_3

[11 Vries, Timmermans, Mereghetti, van Kolck]

Comparison of results

■ Full QCD

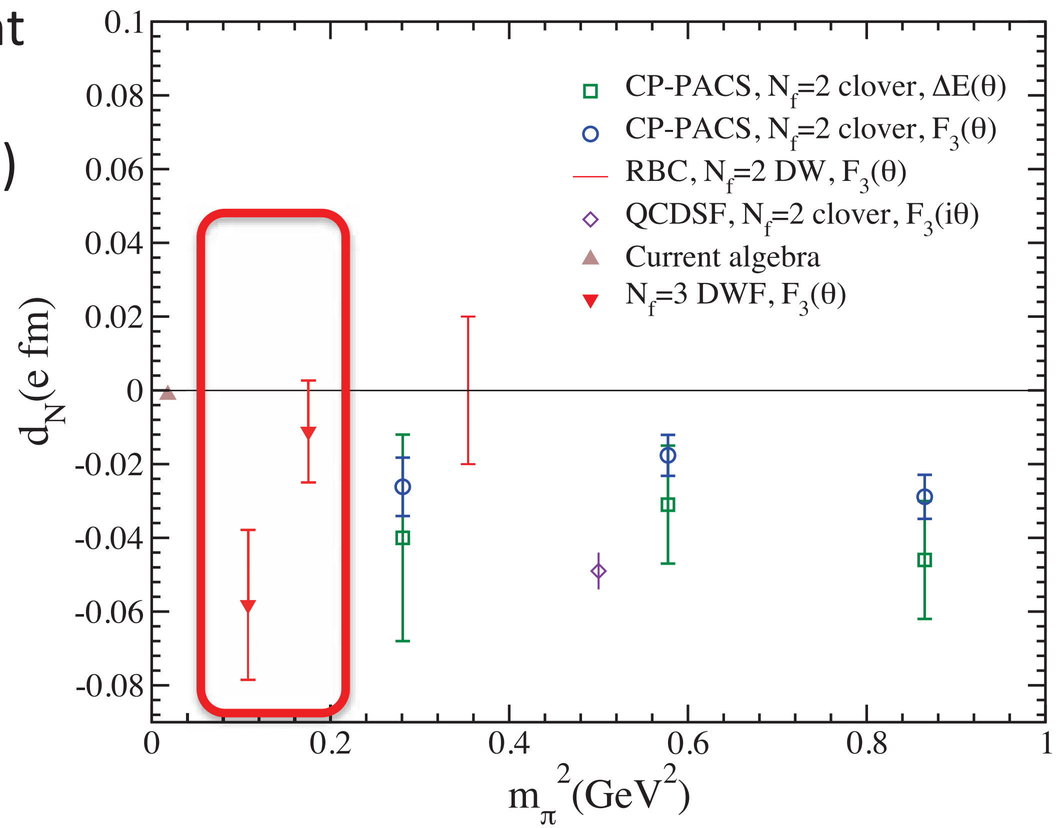
PRELIMINARY

$\Theta = 1$

- Lattice results are consistent within 1σ.
(not include systematic error)
- larger than the results of QCD Sum Rules or ChPT.

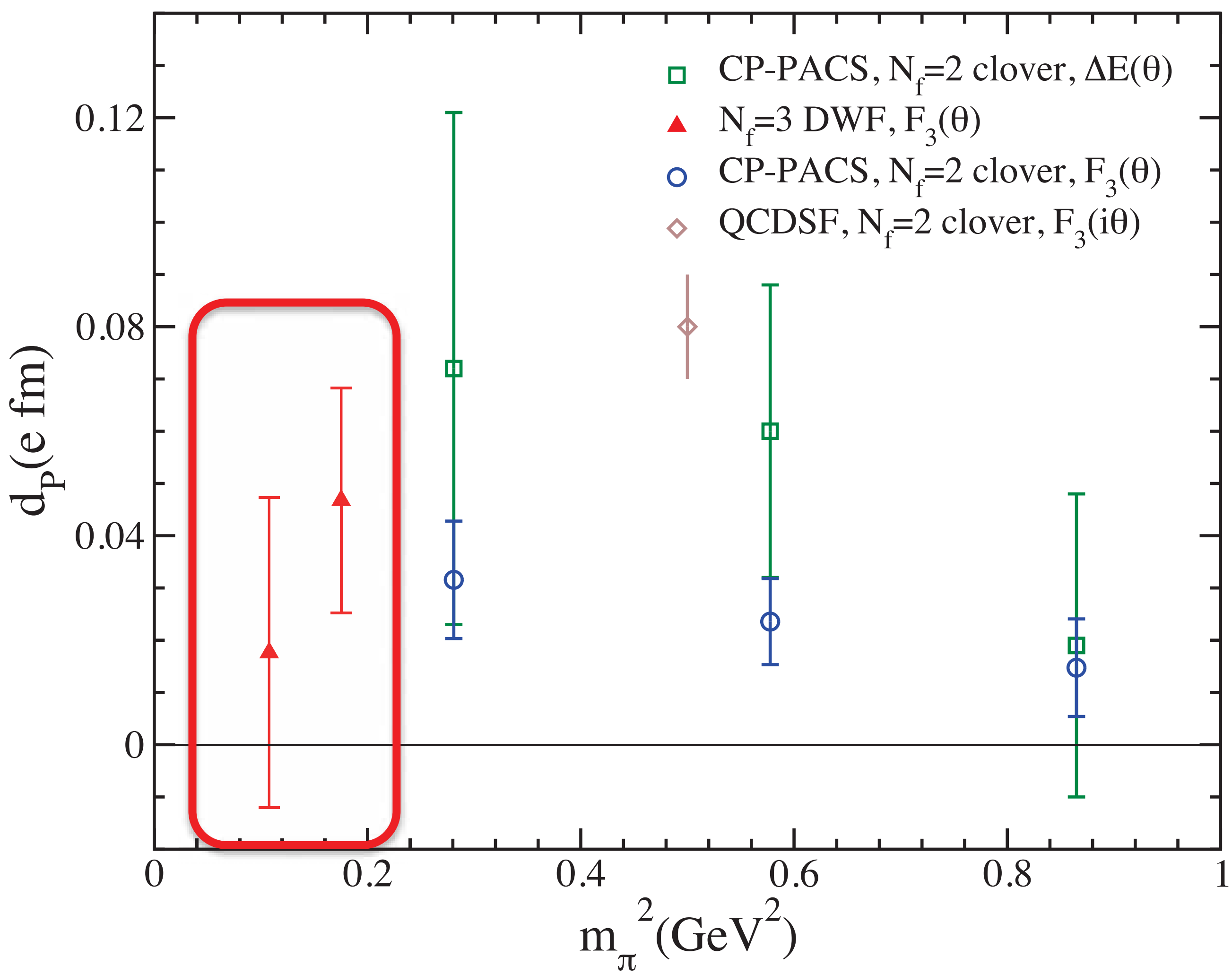
- no obvious quark mass dependence yet

  statistics ?



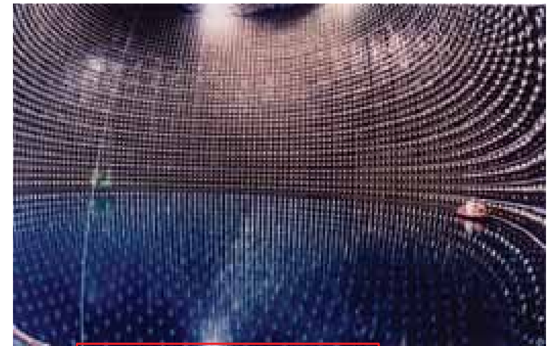
Proton EDM results

PRELIMINARY



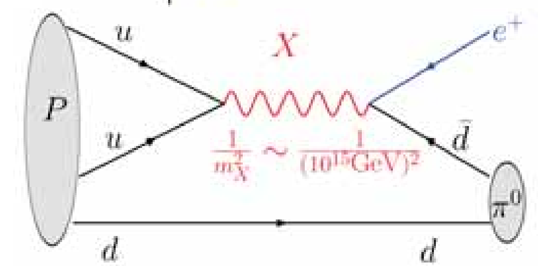
Nucleon calculations for High Energy Physics

- Proton Decay Matrix Elements
  [Y. Aoki, E. Shintani, A. Soni]



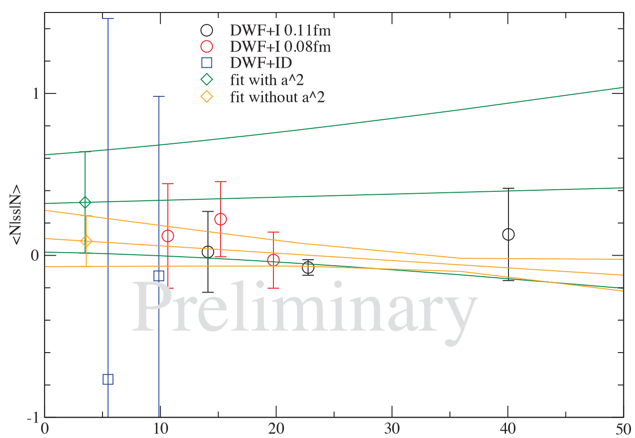
Kamiokande

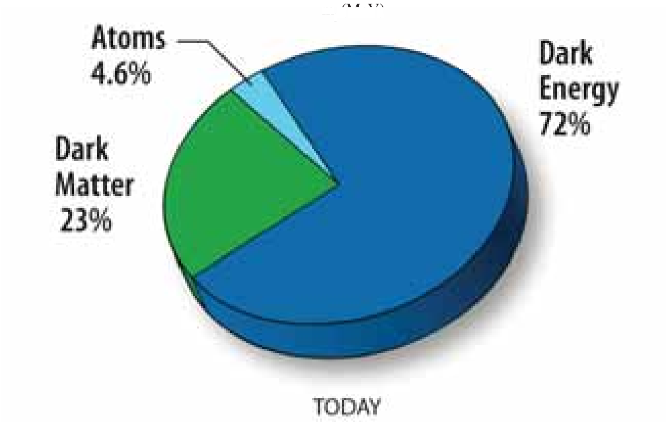
$$\Gamma_P \propto \left| \overline{v_e^c} \langle \pi | (ud)_{R/L} u_L | P \rangle \right|^2$$

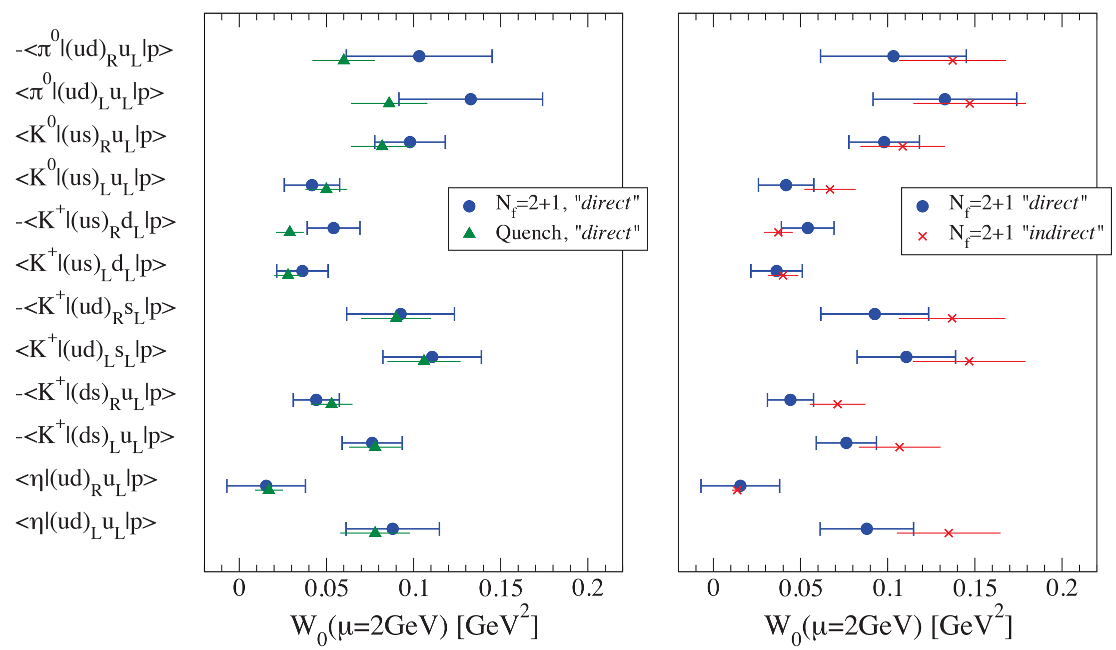


- Strangeness contents in Nucleons for Direct Dirk Matter search
  [C. Jung]

$$\langle N | \bar{s}s | N \rangle$$



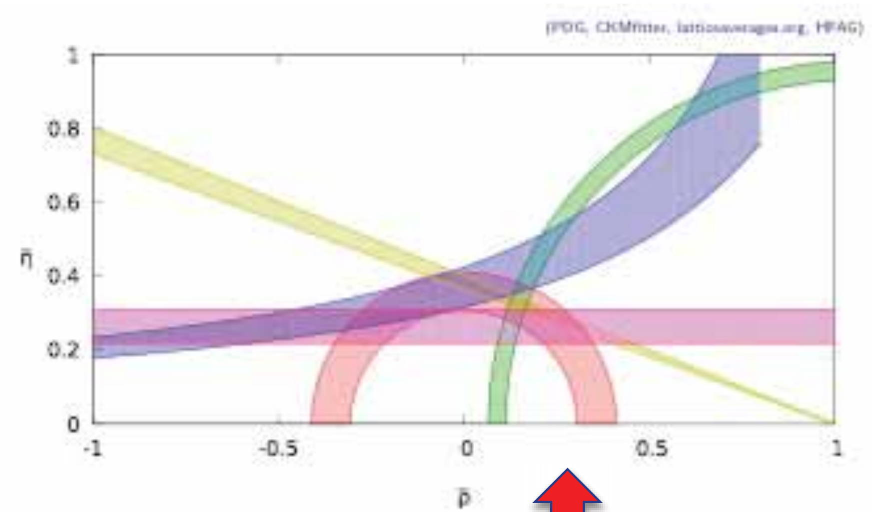


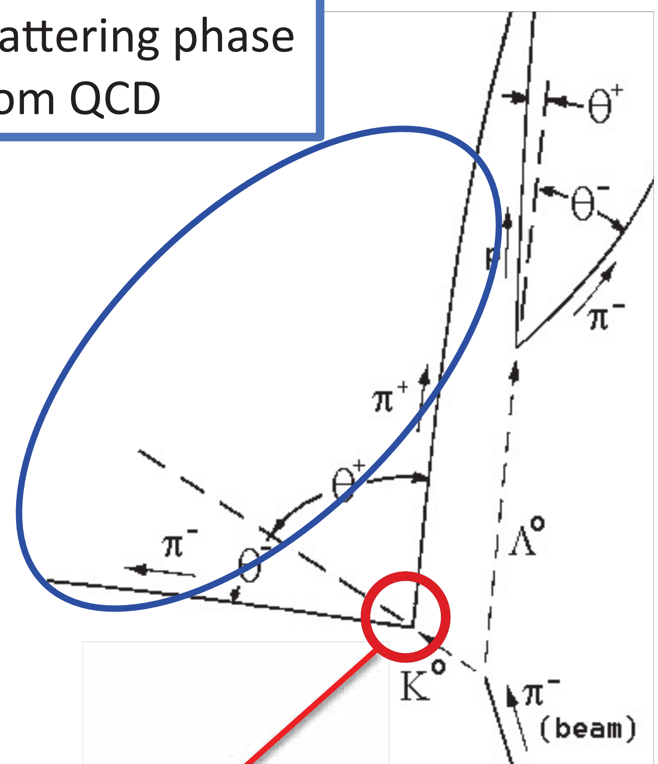


K → π π decay amplitude [RBC/ UKQCD]

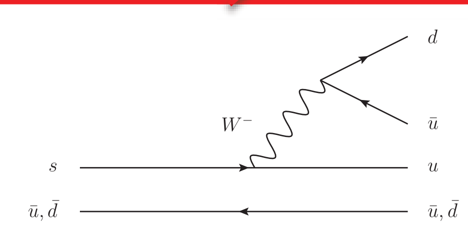
- 40+years awaited theoretical calculation
  [1964 Cronin-Fitch,
  1999 NA48@CERN, KTeV@FNAL]
- Provide a new constraint on CKM Unitarity

π π final state scattering phase from QCD



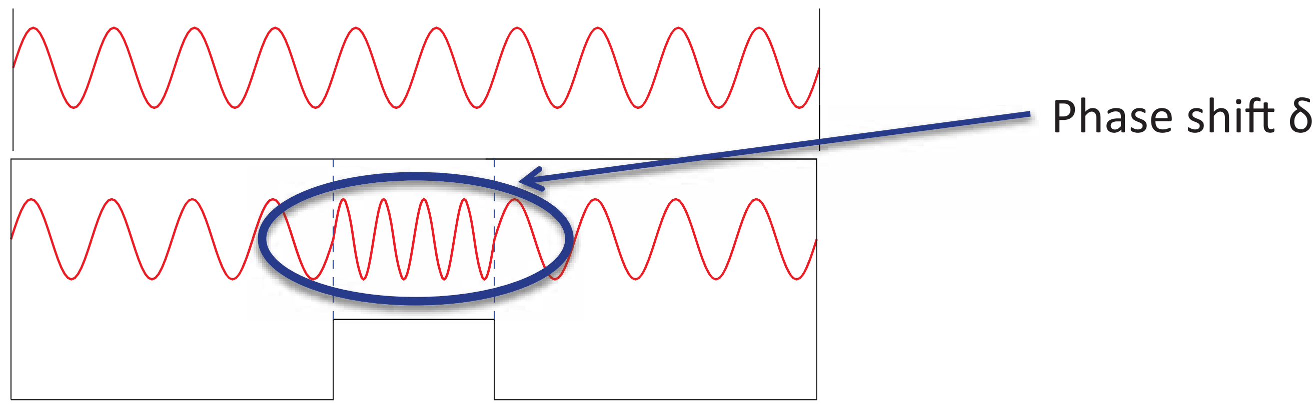


Electro-Weak (CKM) phase
Vus V*ud, Vts V*td

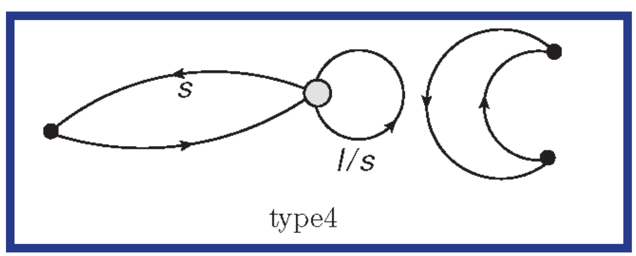


$K \rightarrow \pi\pi$ decay on lattice

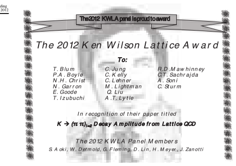
- Relates energy on finite volume $E_{\pi\pi}(V)$ to phase shift δ to obtain complex $\mathrm{Amp}(K \rightarrow \pi\pi) = |A_I| e^{i\delta_I}$ (Luscher, Lellouch-Luscher)



- Momentum of pions are controlled by boundary condition (anti-periodic or G-parity b.c.)
- Mixing and Renormalization of operator is done using non-perturbative renormalization (NPR)
- Chiral Symmetry is curtail
- I=2 channel is under control, I=0 is still a challenge due to disconnected diagrams.



$K \to \pi\pi$ Amplitude, I=2 channel



Lattice: $32^3 \times 64 \times 32$, 2+1 DWF, Iwasaki DSDR, $a^{-1} = 1.375(9)$ GeV, $m_{\pi^+} = 142.9(1.1)$ MeV

	$\mathrm{Re}A_2$	$\mathrm{Im}A_2$
Lattice artifacts	15%	15%
Finite-volume corrections	6.2%	6.8%
Partial quenching	3.5%	1.7%
Renormalization	1.7%	4.7%
Unphysical kinematics	3.0%	0.22%
Derivative of the phase-shift	0.32%	0.32%
Wilson coefficients	7.1%	8.1%
Total	18%	19%

	$\mathrm{Re}(A_2)(10^8/\mathrm{GeV})$	$\mathrm{Im}(A_2)(10^{13}/\mathrm{GeV})$
Lat.	1.436(62)(258)	-6.83(51)(130)
Exp.	1.479(4) (K^+)	(n/a)

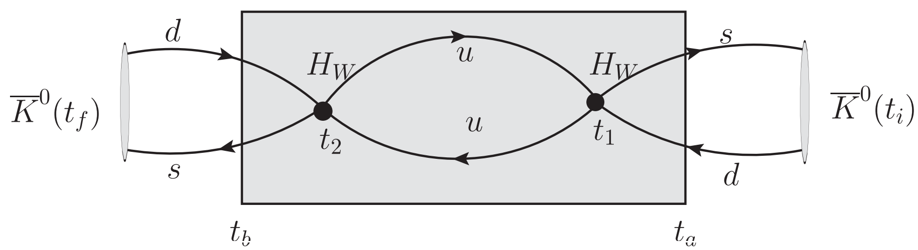
$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{EWP}} = -6.52(49)(124) \times 10^{-4}$

Errors: (stat)(syst)

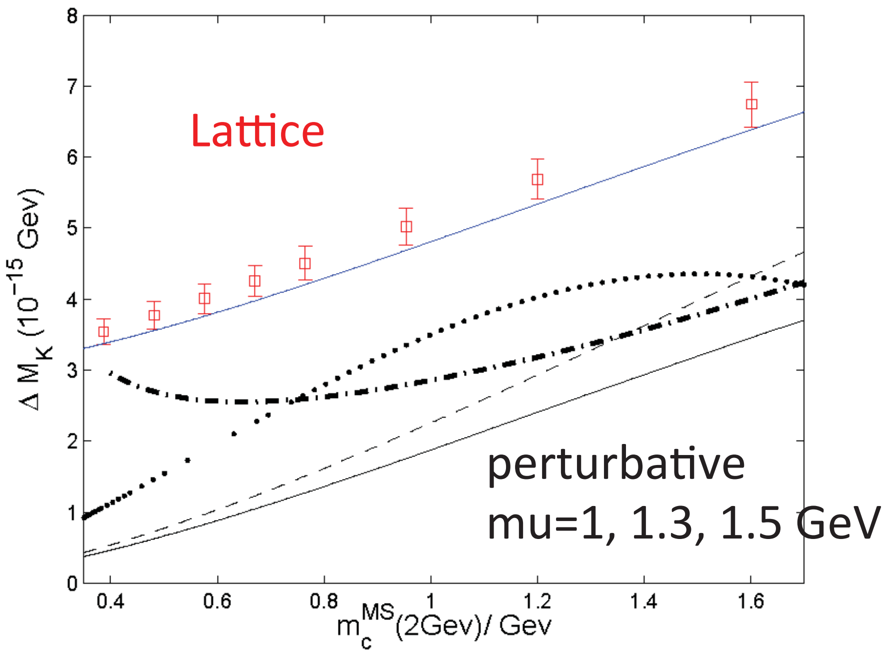
K_L-K_S mass difference

[RBC/ UKQCD, N. Christ, J. Yu et al. arXiv:1212.5931]

- Evaluate K^0-K^0bar amplitude in 2nd order ElectroWeak Hamiltonian : 4pt Green's function



$$\mathscr{A} = \frac{1}{2} \sum_{t_2=t_a}^{t_b} \sum_{t_1=t_a}^{t_b} \langle 0 | T \left\{ \overline{K^0}(t_f) H_W(t_2) H_W(t_1) \overline{K^0}(t_i) \right\} | 0 \rangle .$$



M_K (GeV)	ΔM_K (x 10^{-12} MeV)
563	5.12(24)
707	6.92(39)
918	11.12(94)
1162	20.10(312)

- ΔM_K^{expt} = 3.483(6) 10^{-12} MeV
- Omit disconnected diagrams and Unphysical kinematics, m_π = 421 MeV
- charm quark for GIM cancellation
- 4pt function is useful for the rare Kaon decay : $K \to \pi \nu \nu$

[Blum, TI, Shintani arXiv:1208.4349, arXiv:1212.5542]

Covariant Approximation Averaging (CAA) a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} - \mathcal{O}^{(\mathrm{rest})}$$

unbiased imporved

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

Lattice Symmetry

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(imp)}$ has smaller error
  $O^{(appx)}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply

$$\mathrm{err} \approx C \times \frac{1}{\sqrt{N_{\mathrm{meas}}}}$$

Valence version of Hasenbushing in HMC

Examples of Covariant Approximations (contd.)

- All Mode Averaging **AMA**

  Sloppy CG or Polynomial approximations

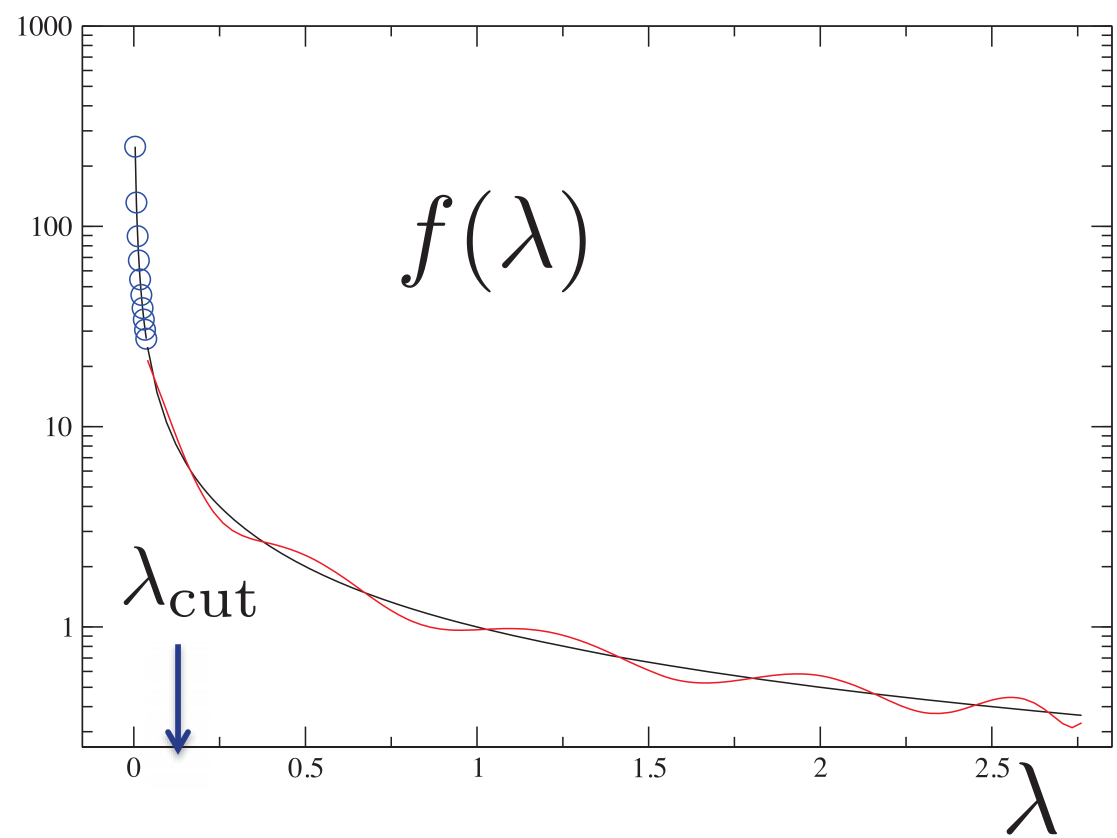
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unneccesary



accuracy control :

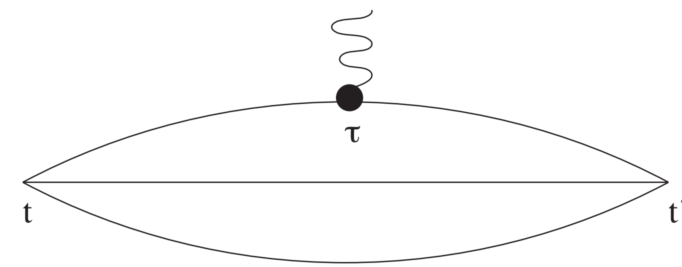
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

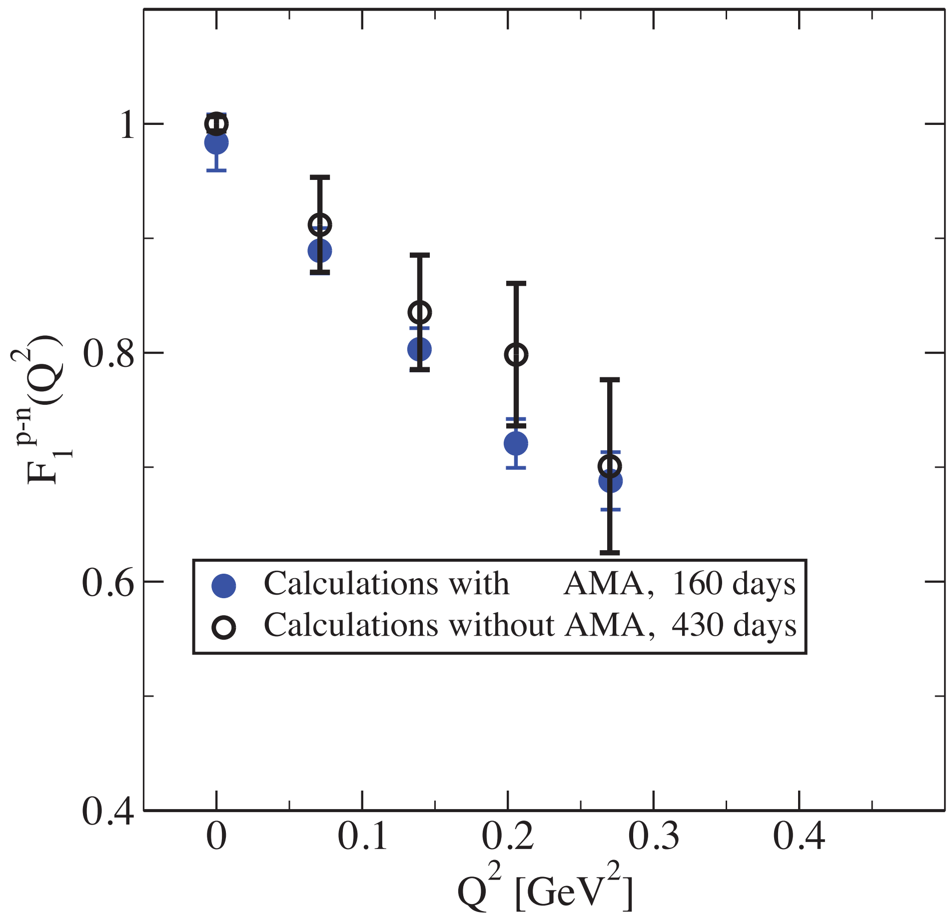
AMA at work

- Target : $V=32^3 \times 64 = (4.6\text{fm})^3 \times 9.6\text{fm}$, Ls=32 Shamir-DWF, a^{-1}=1.37 GeV, Mpi = 170 MeV
- Use Ls=16 Mobius as the approximation
  [Brower, Neff, Orginos, arXiv:1206.5214]

- quark propagator cost on SandyBridge 1024 cores (XSEDE gordon@SDSC)
  - non-deflated CG, r(stop)=1e-8 : ~9,800 iteration, 5.7 hours / prop
  - Implicitly restarting Lanczos of Chebyshev polynomials of even-odd prec operator for 1000 eigenvectors
    [Neff et al. PRD64, 114509 (2001)] : 12 hours
  - deflated CG with 1000 eigenvectors : ~700 iteration, 20 min /prop
  - deflation+sloppy CG, r(stop)=5e-3 : ~125 iteration, 3.2 min /prop
- Multiplicative Cost reduction for General hadrons could combine with {EigCG | AMG} and Distillation:
  x1.2 (Mobius) x 14 (deflation) x 7 (sloppy CG) ~ x 110

AMA at work
[M. Lin]





- $F_1(Q^2)$: tsep = 9 a ~ 1.3 fm

  1 forward + 2 (up and down) seq-props, contraction cost is ~15% of sloppy propagator

- Error bar
  x 2 - 2.7 ~ sqrt(4400/600)
- Total cost reduction upto
  (430 / 160) * (4400/600)
  ~ x19.7

- Note this is still sub-optimal, 4 exact source and without deflation. (would be x30 for 2 exact sources)

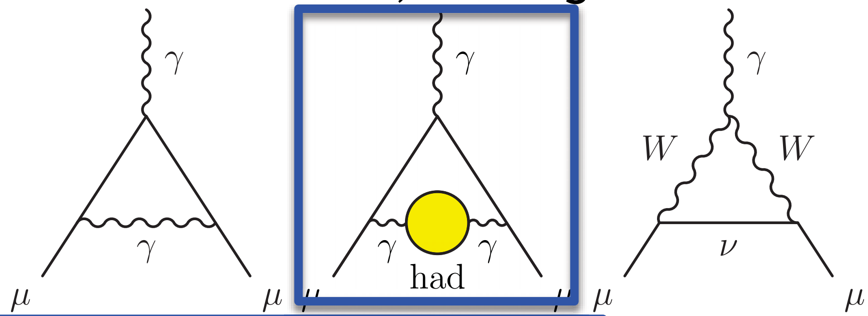
- non-deflated CG, 150 config x 4 sources = 600 measurements :
  5.7 * 3 * 4 * 150 config = 10K hours, 430 days

- AMA : 39 config, 4 exact solves / config (perhaps overkill) , N_G=112 sloppy solves
  => 39 x 112 = 4400 AMA measurements :

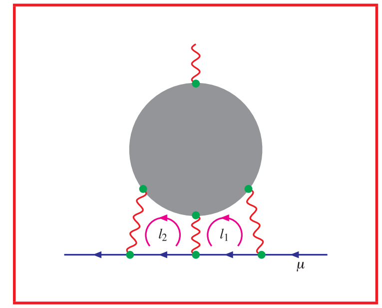
  (5.7 * 3 * 4 + 12 + 0.06 * 3 * 112) * 39 config = 3.9 K hours, 160 days
  4-exact (68%) + Lanczos (12%) + sloppy CG (20%)

muon's anomalous magnetic moment

- One of the most precisely determined numbers, starting from the construction of QED.



Hadronic Vacuum Polarization (HVP)



Hadronic Light-by-Light (HLbL)

$$a_\mu = \frac{g-2}{2} = (116\,592\,089 \pm 54 \pm 33) \times 10^{-11}$$ BNL-E821

[Andreas Hoecker, Tau 2010, arXiv:1012.0055 [hep-ph]]

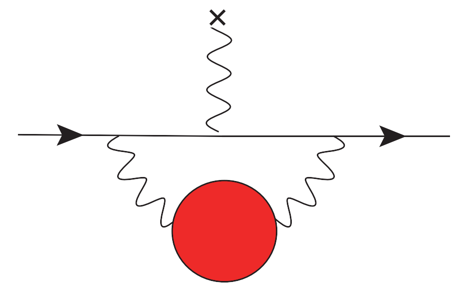
Contribution	Result ($\times 10^{-11}$).
QED (leptons)	116 584 718.09 ± 0.15
HVP (lo)	6 923.± 42
HVP (ho)	-97.9 ± 0.9
HLBL	105.± 26
EW	154.± 2
Total SM	116 591 802 ± $42_{\text{HVP(lo)}}$ ± 26_{HLBL} ± 02 (49_{tot}).

BNL E821
FNAL new g-2
J-PARC

- 287 ± 80 or 3.6σ difference between experiment and SM prediction.

E989 at FNAL is to reduce the total experimental error by, at least, a factor of four over E821, or 0.14 ppm !

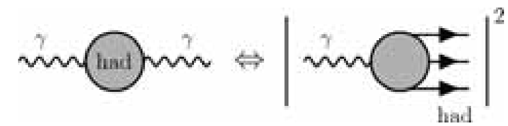
Hadronic Vacuum Polarization



- Currently estimated by σ (e+e-)
  <u>0.6 % accuracy</u>

$$a_\mu^{\rm HVP} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma_{\rm total}(s)$$

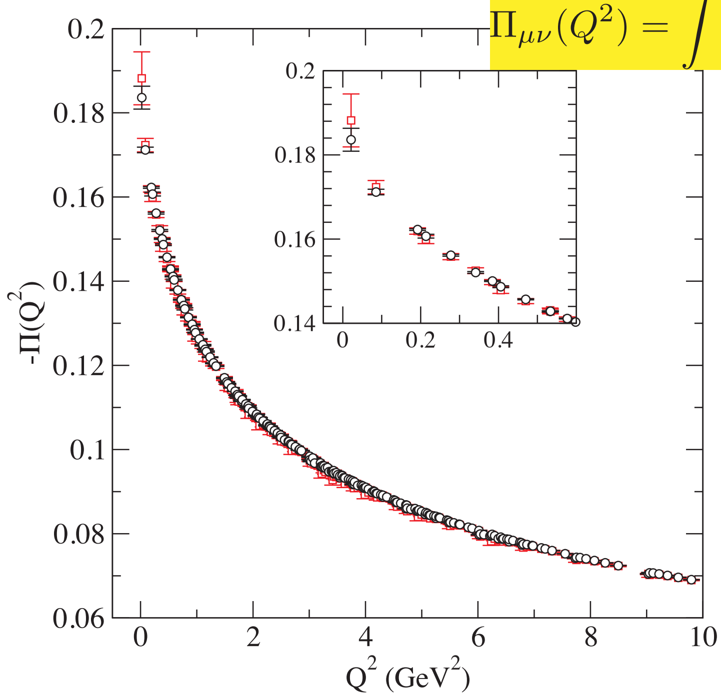
$$a_\mu^{\rm had,LOVP} = (\quad 694.91 \quad \pm 4.27 \quad) \times 10^{-10}$$
$$a_\mu^{\rm had,HOVP} = (\quad -9.84 \quad \pm 0.07 \quad) \times 10^{-10}$$



- Lattice calculation [T.Blum (2003)]

$$\Pi_{\mu\nu}(Q^2) = \int dx \langle V_\mu(x) V_\nu(0)\rangle e^{-q(x-y)}$$

$$a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\Pi(Q^2)$$
$$\Pi_{\mu\nu}(Q) = \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}\right)\Pi(Q^2)$$

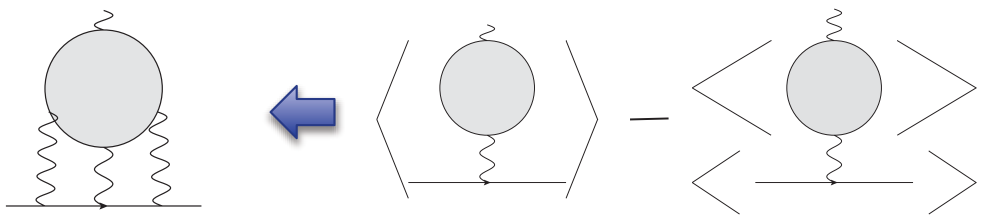


- Use new error reduction technique All Mode Averaging (AMA) <u>x 4- 20 improvements</u> [T.Blum, TI, E. Shintani (2012)]
- accurate π(Q²) at Q²→0 is needed : twisted boundary condition and/or Analytic continuation to Minkowski momentum

to be competitive : O(5-10%) → < O(1%)

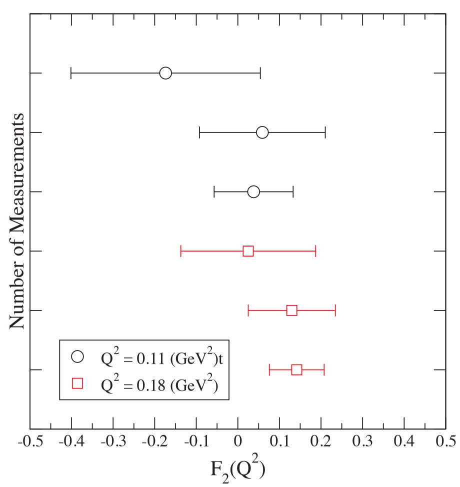
Hadronic Light-by-Light [T. Blum, Hayakawa]

- Compute whole diagram using lattice QCD+QED
- LbL is a part of O(α 3) : need subtraction
  [M. Hayakawa, T.Blum, TI, N. Yamada (2005)]

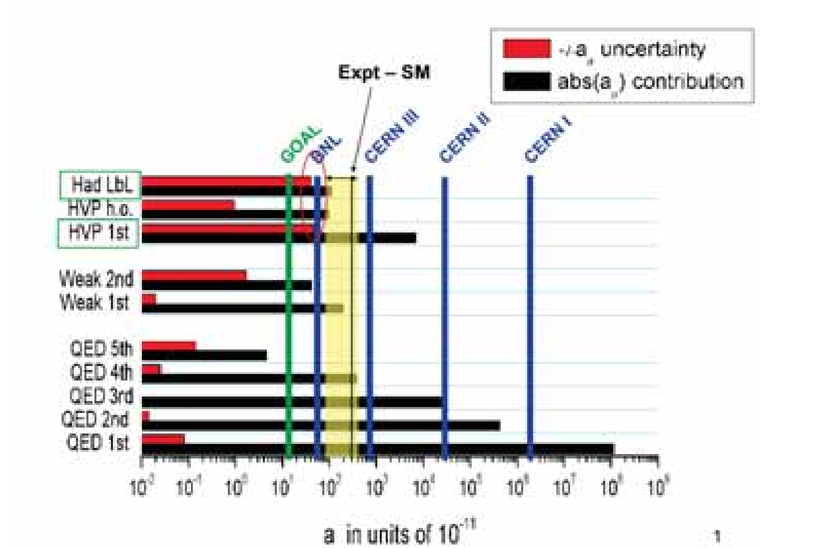


The First signal (preliminary) using AMA

$F_2(Q^2)$ stable with additional measurements (20 → 40 → 80 configs)



24^3 lattice size
$Q^2 = 0.11$ and 0.18 GeV^2
$m_\pi \approx 329$ MeV
$m_\mu \approx 190$ MeV



D. Hertzog, LPNHE Workshop, Paris Feb. 2010

- Very encouraging first results
  order of mag ~ model
- Unphysical mass / momentum
- Disconnected diagrams

Ongoing and Near future plan

- Form factor in DWF configurations
  - RBC/UKQCD collaboration, Chiral symmetry on the lattice

    Reduction of systematic error coming from finite a

    Generate the ensembles including dynamical up-down, strange quarks
  - Large size and small mass

    Control the finite size and chiral extrapolation ($m_\pi \to m_\pi^{phys}$)

    using Mobius fermion [Brower, Neff, Orginos, arXiv:1206.5214]
  - LHPC/RBC/UKQCD collaboration

    An accompanying project to the Nuclear Structure calculation

    [Blum, Syritsyn et al. USQCD proposal 13]

Lattice size	Physical size	Lattice spacing	L_s	Gauge action	Pion mass
$24^3\times64$	2.7 fm^3	0.114 fm	16	Iwasaki	315 -- 615 MeV
$32^3\times64$	2.7 fm^3	0.087 fm	16	Iwasaki	295 -- 397 MeV
$32^3\times64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3\times96$	5.5 fm^3	0.115 fm	24	IW - Mobius	135 MeV
64^3x128	5.6 fm^3	0.087 fm	12	IW-Mbobius	135 MeV

Summary

- New Generation of QCD simulations
- On physics point (M_π=135 MeV) large volume~ (5 fm)^3 QCD ensembles are being generated to avoid systematic errors
- Unprecedented precisions < O(1%)
  EM corrections, EM Polarizabilities,
  quark masses, decay constants, B_K, B & D,
  $K \to (\pi\pi)_{I=2}$, (g-2) HVP, Proton decay, ...
- Unprecedented physics computations
  $K \to (\pi\pi)_{I=0}$, $\Delta M(K_L - K_S)$, Kaon rare decays,
  (g-2) LbL, EDM, Hadronic Parity Violation,....
- Enabling technologies
  New resources : QCDCQ, K computer, GPU, ... x 20
  New algorithms : AMA, A2A, Mobius, EigCG, ... x 20

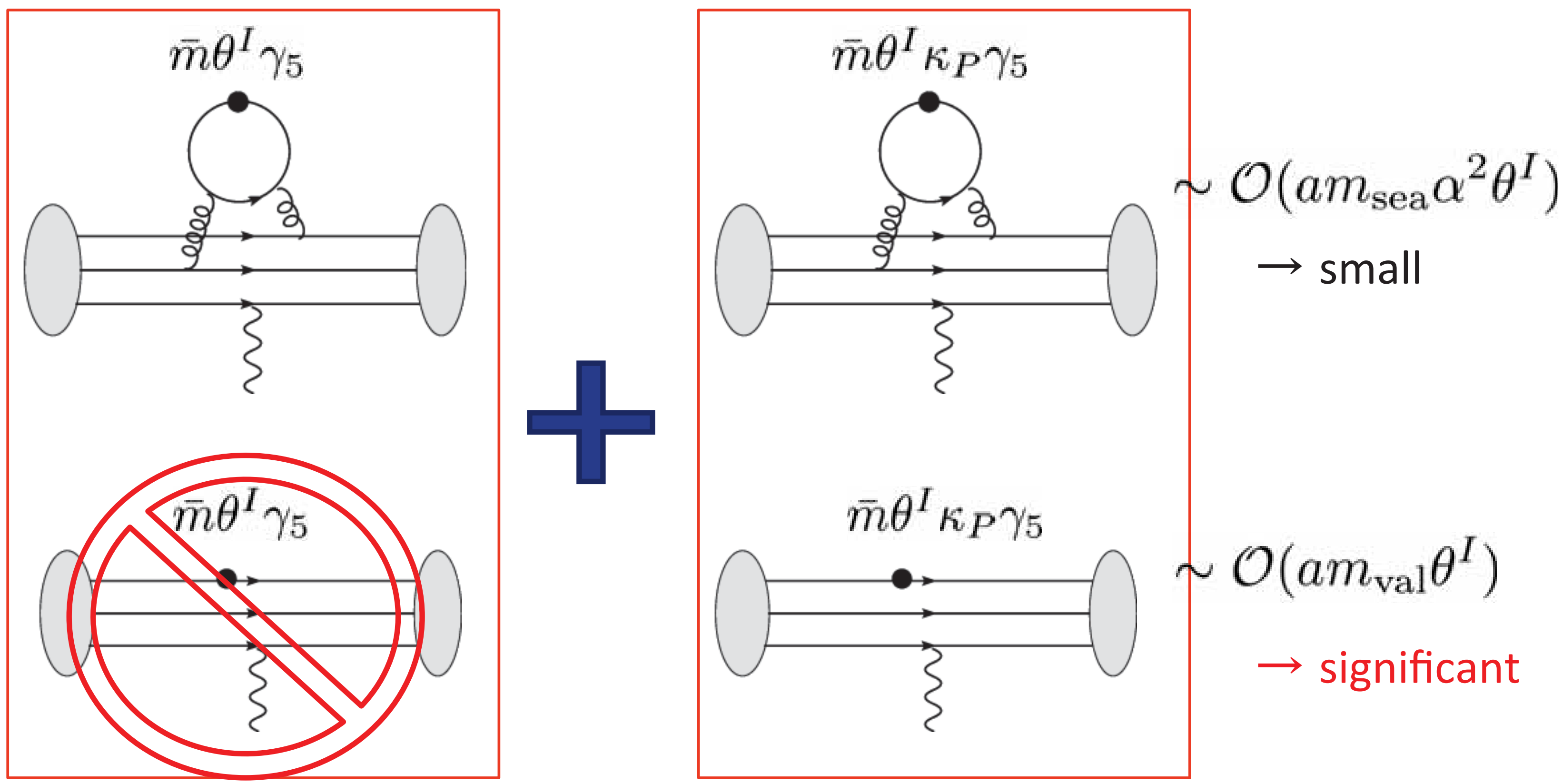
The issue of chiral breaking

Izubuchi(07), Horsley et al. (08)

- Full QCD with Wilson fermion O(a) chiral breaking

Fermionic insertion of imaginary theta should be changed by Wilson term:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2 \to \mathcal{L}_\theta^W = \bar{m}(1+\kappa_P)\theta^I \bar{q}\gamma_5 q,\ \kappa_P \sim \mathcal{O}(a) : \text{renom. const.}$$



Cf. discussion in Aoki, Gocksh, Manohar,Sharpe (1990)

Cost comparison for test cases

- x 16 for DWF Nucleon mass (M_{PS}=330MeV, 3fm)
- x 2- 20 for AsqTad HVP (MPS=470 MeV, 5 fm)
- should be better for lighter mass & larger volume !

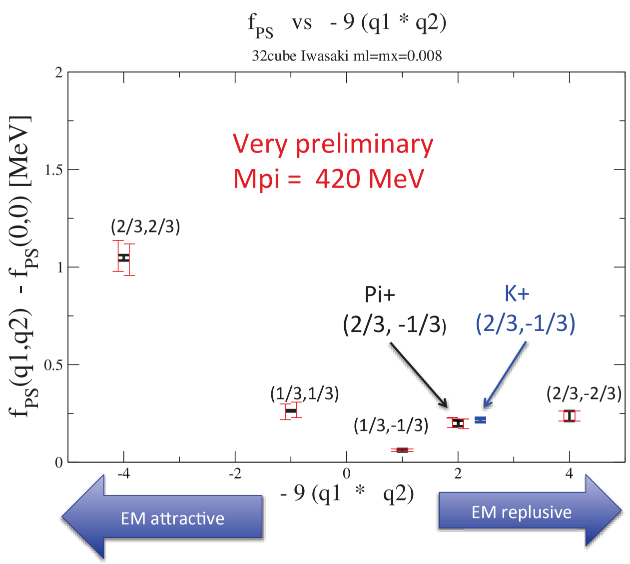
	$N_{\rm conf}$	$N_{\rm meas}$	LM	$\mathcal{O}$	$\mathcal{O}_G^{(\rm appx)}$	Tot.	scaled cost	
m_N	$m = 0.005$, 400 LM						gauss	pt
AMA	110	1	213	18	91+23	350	0.063	0.065
LMA	110	1	213	18	23	254	0.279	0.265
Ref. [2]	932	4	-	3728	-	3728[a]	1	1
	$m = 0.01$, 180 LM							
AMA	158	1	297	74	300+22	693	0.203	0.214
LMA	158	1	297	74	22	393	0.699	0.937
Ref. [2]	356	4	-	1424	-	1424	1	1
HVP	$m = 0.0036$, 1400 LM						max	min
AMA	20	1	96	11	504+420	1031	0.387	0.050
LMA	20	1	96	11	420	527	10.3	3.56
Ref. [1]	292	2	-	584	-	584	1	1

- ✓ x 20 is observed more to expect
- ✓ DIPLOMATIC ALGORITHM
- ✓ Other type "approximations"
  Mobius fermion

QCD+QED simulation

[T. Blum et al.]

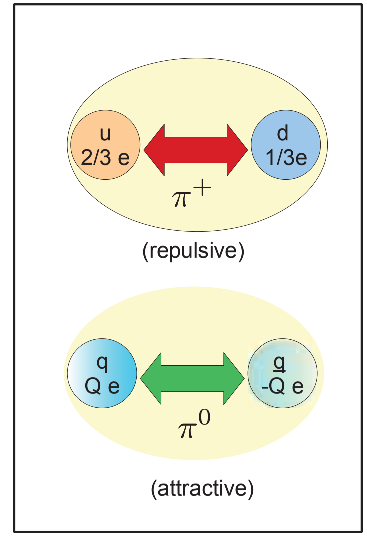
- EM effects on PS decay



- Statistically well resolved by +e/-e averaging.
- c.f. [Bijnens Danielsson 2006]

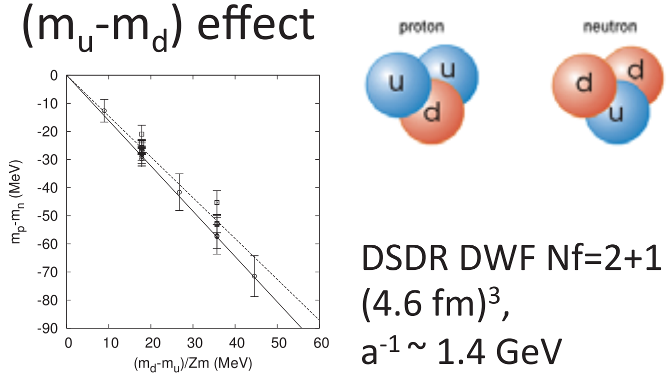
$$f_{\pi^+,\mathrm{NLO}}/F_0 = 0.0039$$
$$f_{K^+,\mathrm{NLO}}/F_0 = 0.0056$$

- EM turned on, but mu = md
- Iwasaki-DWF Nf=2+1,
- (2.7 fm)^3, a^{-1} ~ 2.3 GeV



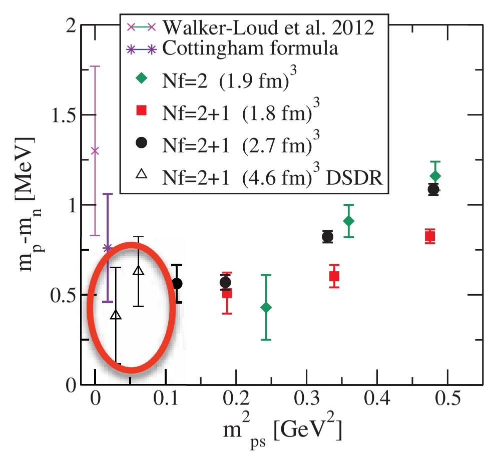
- Proton / Neutron mass difference

(m_u-m_d) effect



DSDR DWF Nf=2+1
(4.6 fm)^3,
a^{-1} ~ 1.4 GeV

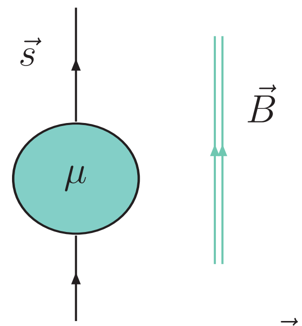
EM effect



	$m_u - m_d$	EM
NPLQCD	2.26(72)	
BLUM	2.51(71)	0.54(24)
RM123	2.80(70)	
QCDSF-UKQCD	3.13(77)	
	2.68(35)	0.54(24)

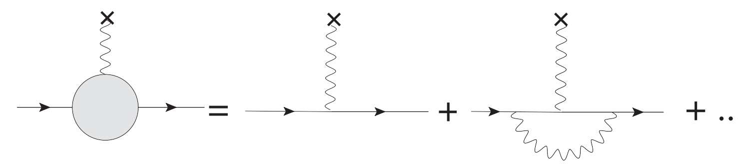
$$\Longrightarrow M_N - M_p| = 2.14(42)\ \ \mathrm{MeV}$$
(experiment: 1.2933321(4) MeV)

Muon anomalous magnetic moment
[T. Blum, T. Ishikawa, E. Shintani et al.]

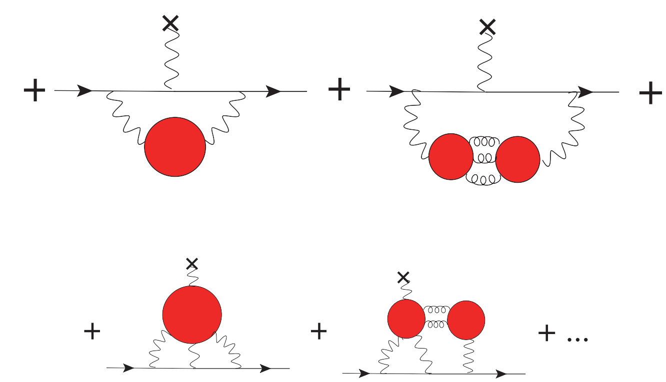


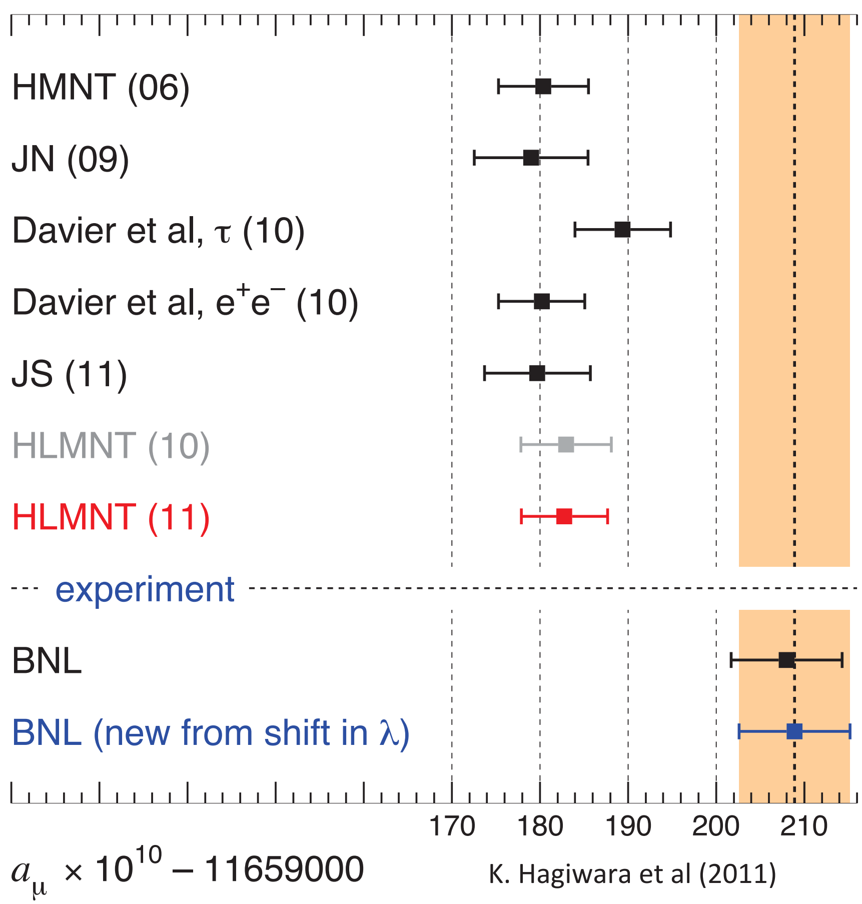
$E = -g_\mu \frac{e}{2m_\mu} \vec{s} \cdot \vec{B}$

$$a_l = \frac{g_l - 2}{2}$$



1/α = 137.035 999 166 (34) [Aoyama et al. (12)]





K. Hagiwara et al (2011)

$$a_\mu^{\rm SM} = (11\ 659\ 182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{\rm QED} = (11\ 658\ 471.808 \pm 0.015) \times 10^{-10}$$
$$a_\mu^{\rm EW} = (15.4 \pm 0.2) \times 10^{-10}$$
$$a_\mu^{\rm had,LOVP} = (694.91 \pm 4.27) \times 10^{-10}$$
$$a_\mu^{\rm had,HOVP} = (-9.84 \pm 0.07) \times 10^{-10}$$
$$a_\mu^{\rm had,lbl} = (10.5 \pm 2.6) \times 10^{-10}$$

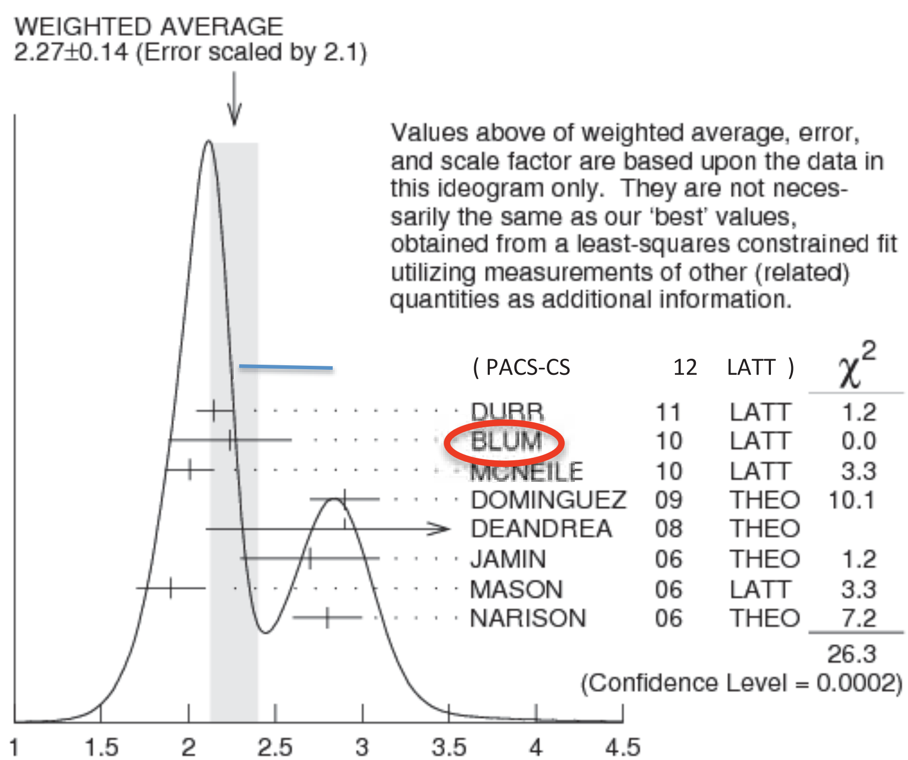
FNAL E989, J-PARC
aim for x 4-5 accuracy

$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = (26.1 \pm 8.0) \times 10^{-10}$ ~3 σ discrepancy

up quark mass

T. Blum et al. Phys.Rev. D82 (2010) 094508
T. Ishikawa et al. Phys.Rev.Lett. 109 (2012) 072002

- input : experimental masses on $\pi^{\pm}$, $K^{\pm}$, K^{0}
- Lattice QCD+QED simulation to solve for masses of up, down, strange quarks taking into account quark's electric charges

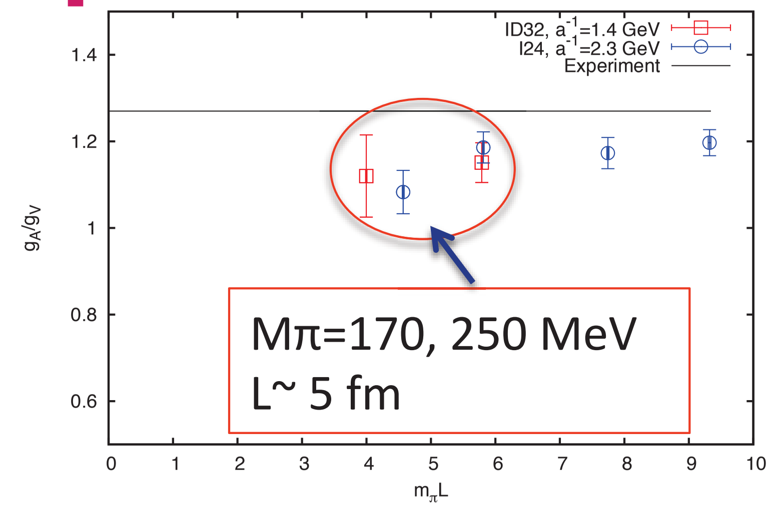


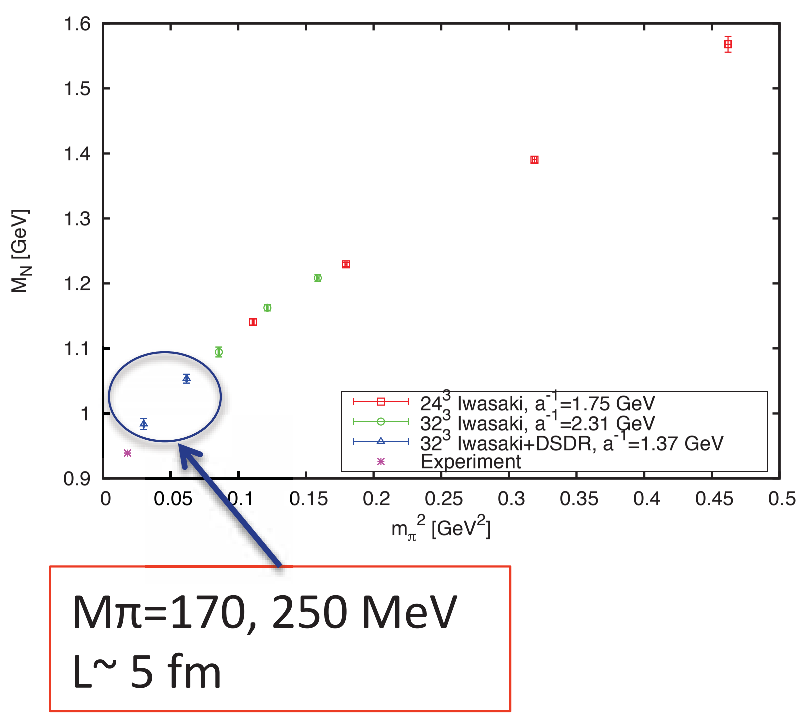
PDG 2012 average
m(up) = 2.27± 0.14 MeV
MSbar at 2 GeV

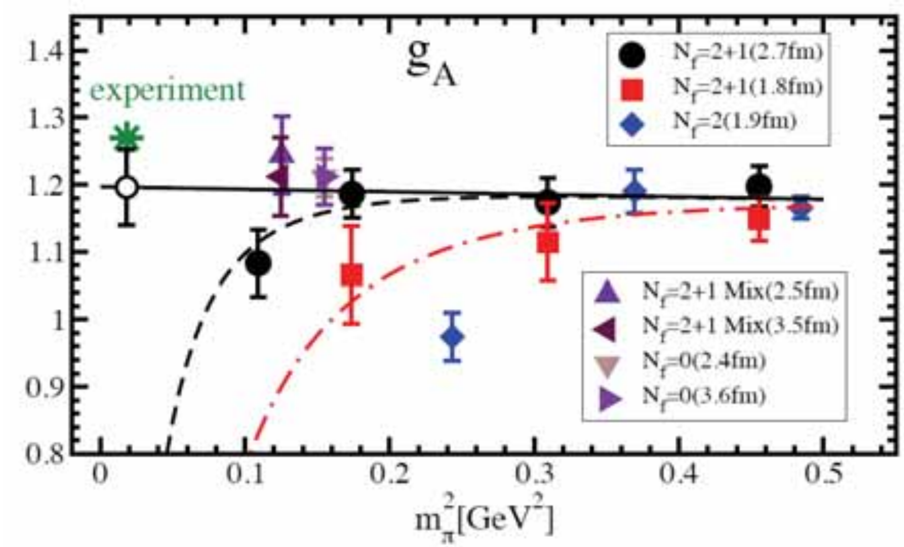
Nucleon calclations

[RBC + LHP]

- Nucleon axial charge g_A
  - Finite Volume Effect ?
  - Excited contamination ?
- Nucleon form factors
- Nucleon (generalized) structure functions
- Origin of Nucleon Spin
  [Electron Ion Colliders, BNL, JLAB]

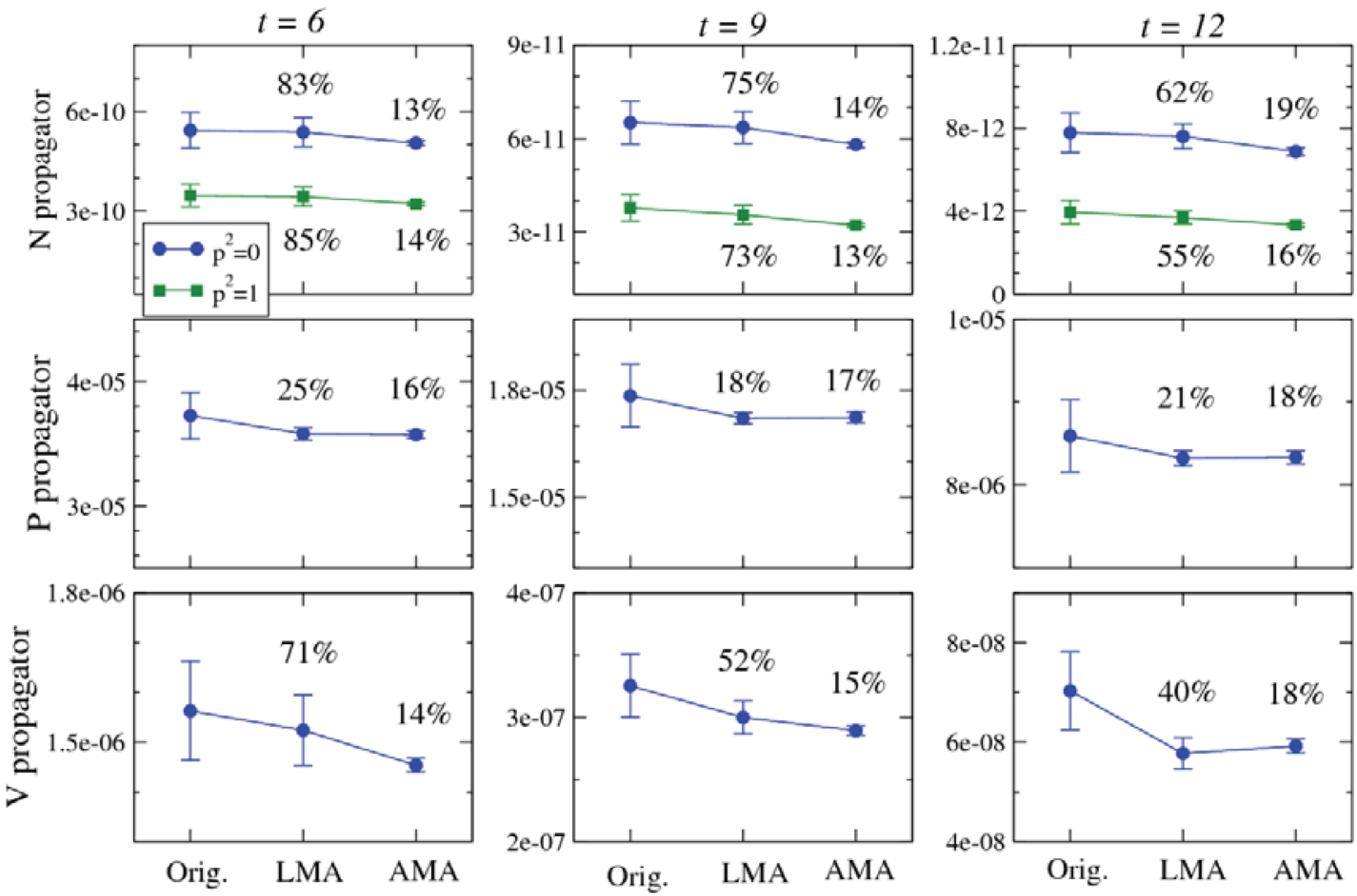






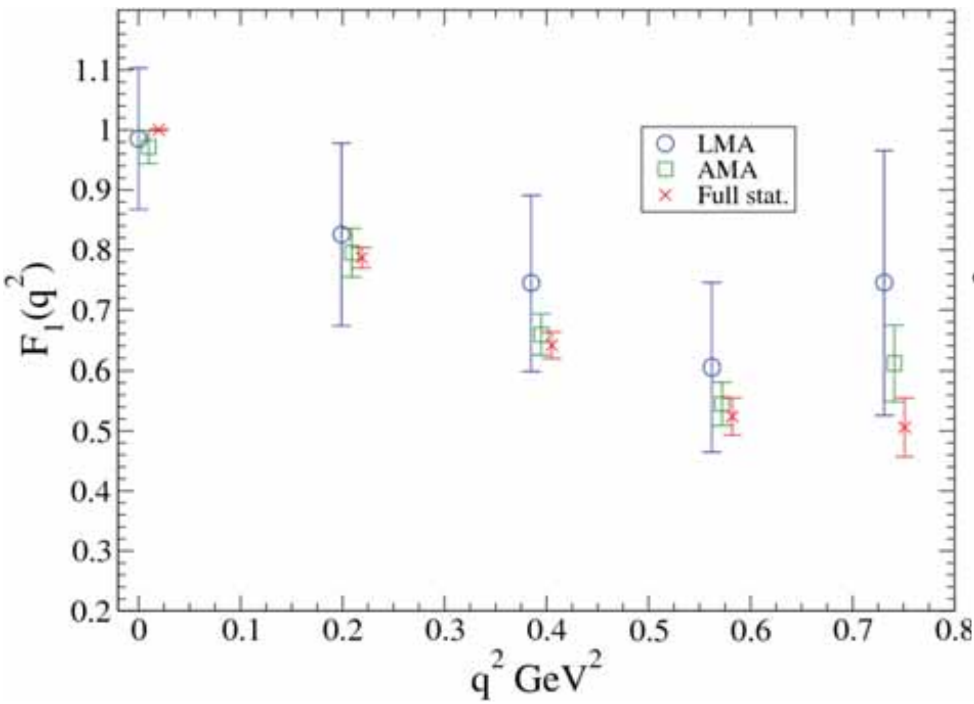
- Advantages of chiral lattice quark
- More demanding calculations
- limited by statistical error

AMA results for hadron 2pt functions [E. Shintani]

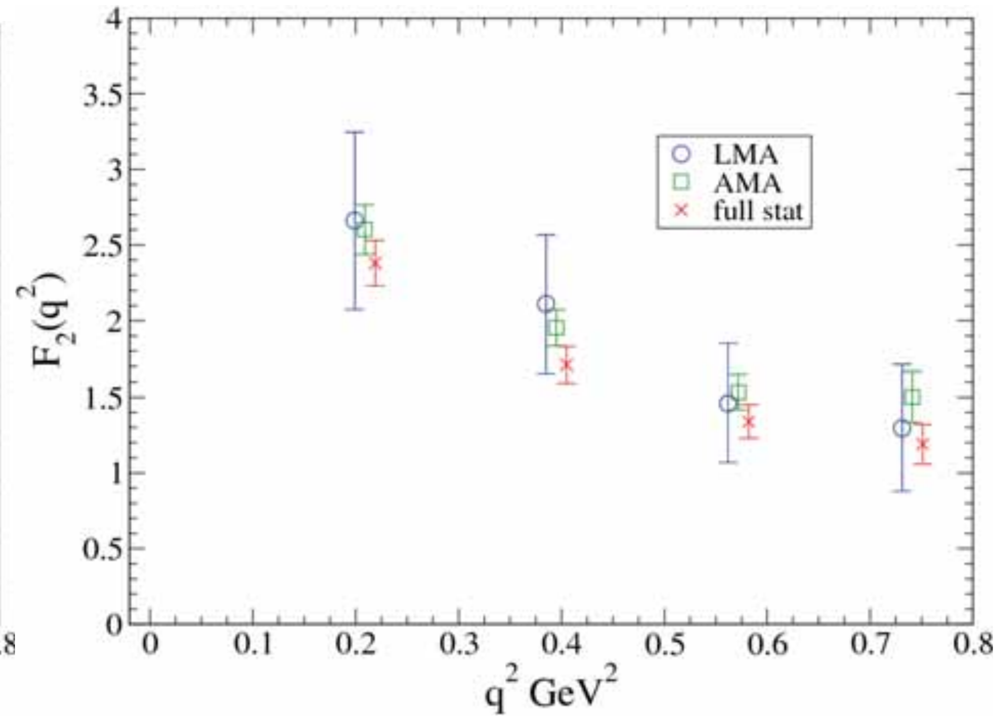


Comparison of isovector $F_{1,2}$ [E. Shintani]

m=0.01



m=0.01

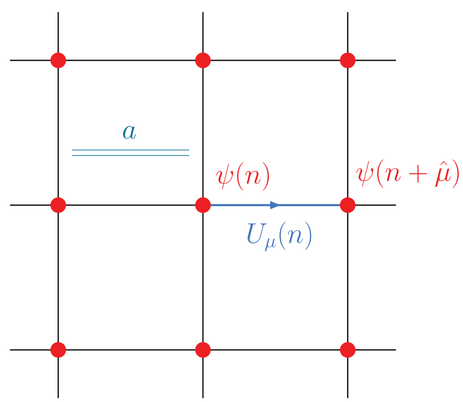


- Results are well consistent with full statistics.
- Statistical error is much reduced in AMA rather than LMA.
- Compared to full statistics, AMA results (m=0.01) have still 1.2 -- 1.5 times larger statistical error (except for $F_1(0)$).
- This may be due to correlation between different source points.

Lattice Gauge Theory

- Analysis of Quantum Field Theory such as Quantum Chromo Dynamics, needs non-perturbative calculation.

  $\Psi(x), A_\mu(x), x \in \mathcal{R}^4$: *continuous infinity*
  quantum divergences: needs regularization and renormalization



- Discretized Euclidean space-time
- lattice spacing $a \sim 0.1$ fm
  (UV cut-off $|p| \leq \pi/a$)

- $\psi(n)$: Fermion field (Grassmann number)
- $U_\mu(n)$: Gauge field

- Feynman's path integral for Huge dimensional variables $32^3 \times 64 \sim 150$M
  Number of states (for simplest 4^4 Ising) $2^{4^4} \sim 10^{77}$ needs more than 10^{35} years !

Lattice Gauge Theory Receipt

- QCD vacuum ensemble generation $\sim$ Accelerator

  - choice of gauge / sea quark actions
  - Algorithms / Machines **capability machines**
  - for each parameters $(a^{-1}, V, m^{\text{sea}})$

- Physical observable measurements $\sim$ Detector
  - valence quark propagators (low eigenvectors), m_f
  - Hadron n-point green's functions, matrix elements

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \mathcal{D}\bar{q}_i \mathcal{D}q_i \mathcal{O} e^{-S_{\text{LGT}}} / \langle 1 \rangle$$

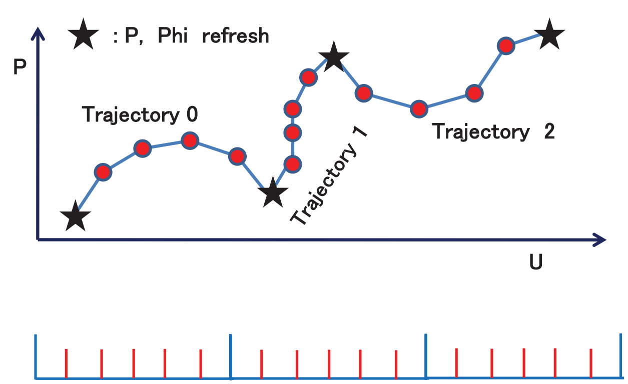
  - Renormalize and Chiral/Volume/continuum extrapolations
  - Algorithms / Machines **capacity machines**

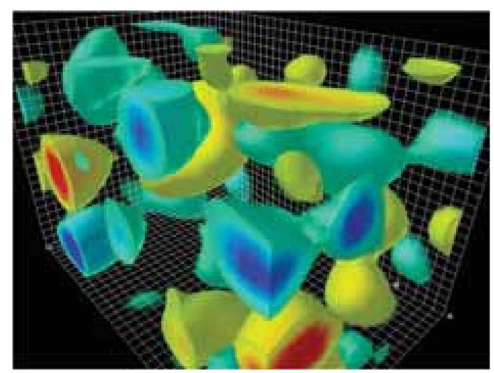
$\Longrightarrow$ The final answers

Hybrid Monte Carlo (LGT's "Accelerator")

- Monte Carlo to Sample Important configurations of QCD action $e^{-S_{\text{QCD}}}$
- Accumulate samples of QCD vacuum, typically $\mathcal{O}(100) \sim \mathcal{O}(1,000)$ files of gauge configuration $U_\mu(n)$ on disk (1 $\sim$ 10 GB/conf).
- By solving a classical QCD, with an occasional stochastic "hit": exactly $\propto e^{-S_{\text{QCD}}}$
- Must generate sequentially $\{U_\mu^{(0)} \to U_\mu^{(1)} \to \cdots\}$, which needs capable machines.

$$\text{Prob}(U_\mu) \propto \det D_{u,d,s}[U] e^{-S_g}$$





[D. Leinweber]

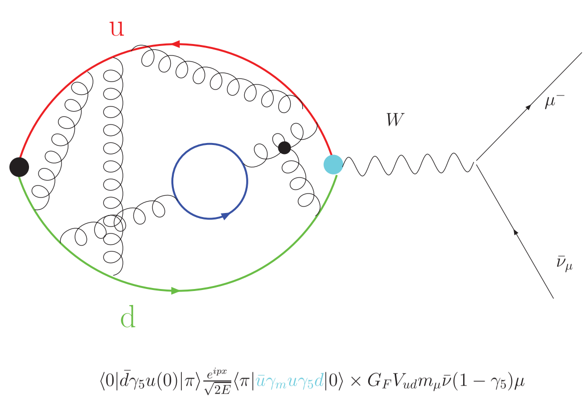
- RHMC for odd flavor [Clark Kennedy]
- Solve short (long) modes more (less) frequently [Hasenbush's trick]

Physics measurements "Detectors"

- Measurements physical observables on the vacuum ensemble.

$$\langle \mathcal{O} \rangle = \int \underline{\mathcal{D}U_\mu \, \text{Prob}[U_\mu]} \times \mathcal{O}[U_\mu]$$

- Could do Analysis on many configurations independently (trivial parallel jobs) $\longrightarrow$ could also use PC Clusters
- We made hadron operator (EW operators) from quark, and let the quark propagates on each of the generated QCD configuration (by solving the Dirac Eq)
- Obtain hadron mass or QCD matrix elements of operators



$$\mathcal{M}(\pi \to \mu\overline{\nu}) \sim i f_\pi q_\mu \times G_F V_{ud} m_\mu (\overline{\nu}\mu)_L$$

$$= \langle \pi(q)|\bar{u}\gamma_\mu\gamma_5 d(0)|0\rangle \times G_F V_{ud} m_\mu (\overline{\nu}\mu)_L$$

Multiple timestep in HMC

- Multiple time steps in MD integrators
- Sexton & Weingarten trick
- Hasenbusch trick : introduce intermediate mass

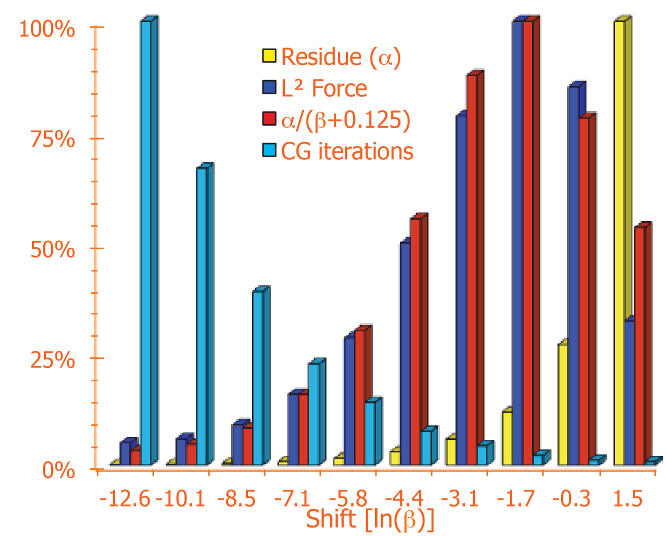
$$\det[D(m)] = \overbrace{\det[D(m_I)]}^{\text{cheap mode}} \times \overbrace{\det\left[D(m)D(m_I)^{-1}\right]}^{\text{expensive mode}}$$

- Clark & Kennedy RHMC (quotient force term)

Berlin Wall was torn down
by Smart Work Sharings

Similar tricks for valence ?

A. Kennedy 06



Unbiasness proof

- Consider a element g of lattice symmetry G e.g. $x_\mu \to x + \Delta x_\mu^{(g)}$
- transformation of fields

$$U_\mu(x) \to U_\mu^g(x) = U_\mu(x - \Delta x^{(g)})$$

$$\begin{aligned}\mathcal{O}[U_\mu] \to & \mathcal{O}^g[U_\mu^g](x_1, x_2, \cdots, x_n) \\ & = \mathcal{O}[U_\mu^g](x_1 - \Delta x^{(g)} x, x_2 - \Delta x^{(g)} x, \cdots, x_n - \Delta x^{(g)} x),\end{aligned}$$

- Observable (and its approximation) is called to have covariance under g iff

$$\mathcal{O}^g[U_\mu^g](x_1, x_2, \cdots, x_n) = \mathcal{O}[U_\mu](x_1, x_2, \cdots, x_n)$$

or, more explicitly,

$$\mathcal{O}[U_\mu^g](x_1 - \Delta x^{(g)}, x_2 - \Delta x^{(g)}, \cdots, x_n - \Delta x^{(g)}) = \mathcal{O}[U_\mu](x_1, x_2, \cdots, x_n) \quad .$$

- When g is a symmetry of lattice, and $O^{(appx)}$ is covariant $\langle \mathcal{O}^g \rangle = \langle \mathcal{O} \rangle$

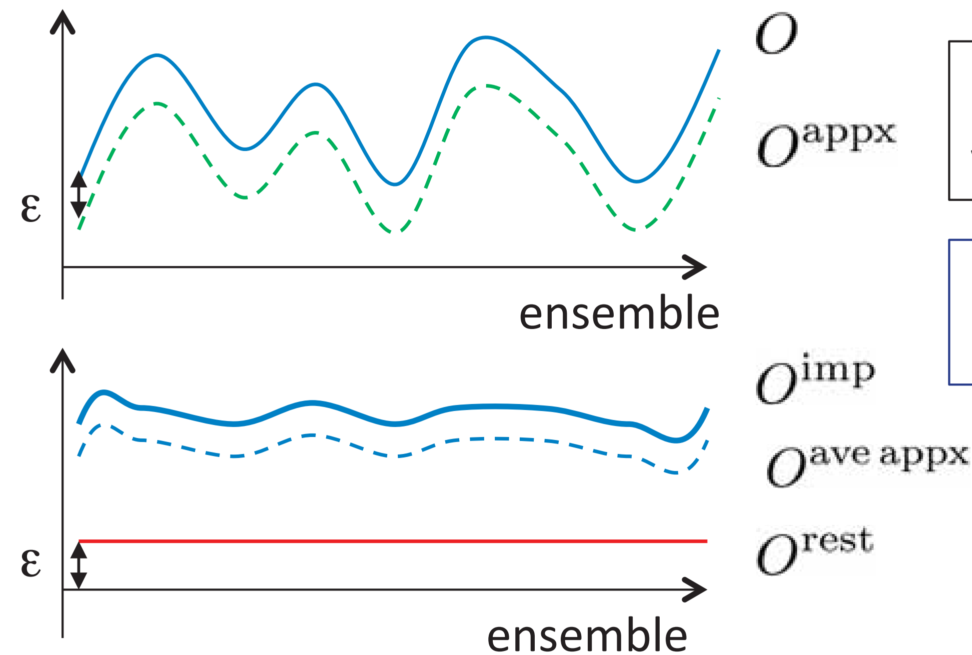
$$\mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

$$\langle \mathcal{O}^{\mathrm{imp}} \rangle = \langle \mathcal{O} \rangle$$

AMA : a smart work sharing

- Ideal approximation



O^{appx} is ***strongly correlated*** with original one.

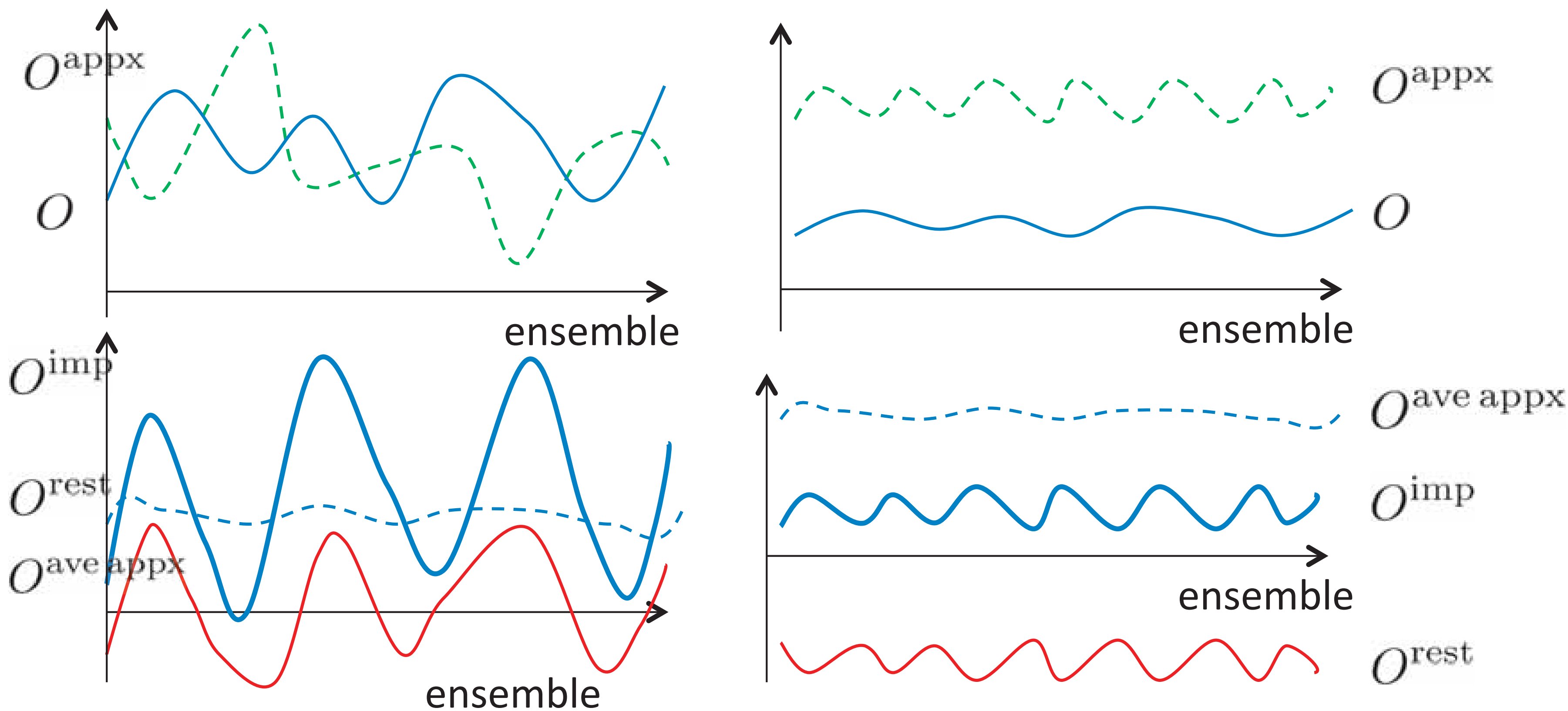
R(corr) b/w O and $O^{(appx)}$ needs to be larger than 0.5 [C. Lehner]

$$\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_g}$$

- ε, accuracy of approximation should be smaller than $O^{\mathrm{ave\ appx}}$
- ΔO^{rest} which is statistical error of O^{rest} depends on the strength of correlation.
- The computational cost of O^{appx} should be much smaller than original.

AMA : not working

- Nightmare case
  - Anti-correlated or bad approximation



$$\mathrm{err}^{\mathrm{imp}} \gg \mathrm{err}$$

Examples of covariant approximations

- Low mode approximation used in the Low Mode Averaging (**LMA**)

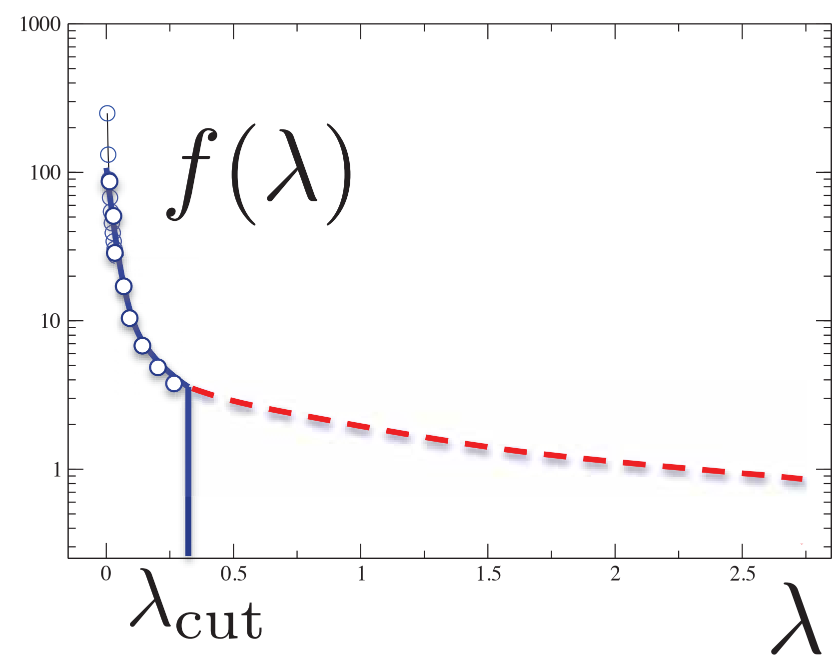
  L. Giusti et al (2004), see also T. DeGrand et al. (2004)

  accuracy control : # of eigen mode

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \frac{1}{\lambda}\theta(\lambda_{\mathrm{cut}} - |\lambda|)$$



Deflation using low eigenmodes from Lanczos [Neff et al, JLQCD]

- 4D even/odd preconditioning

$$D_{DW} = \begin{pmatrix} M_5 & K(M_4)_{eo} \\ K(M_4)_{oe} & M_5 \end{pmatrix}$$

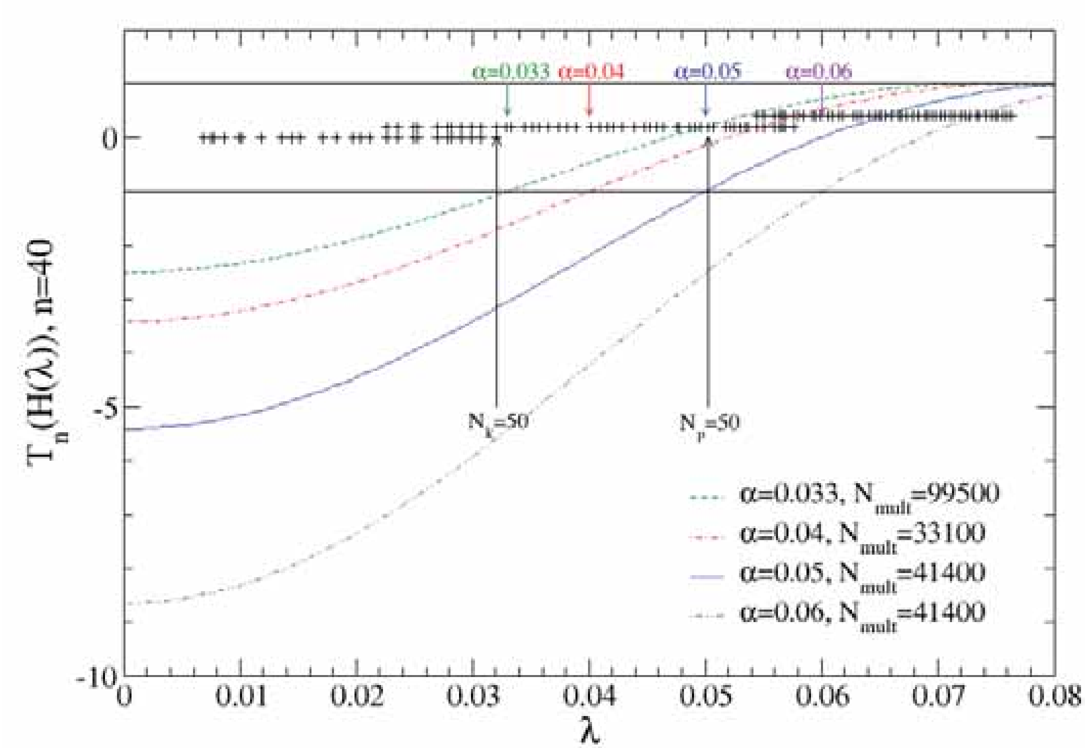
[R. Arthur]

$$D_{DW}^{-1} = \begin{pmatrix} 1 & 0 \\ -KM_5^{-1}(M_4)_{oe} & M_5^{-1} \end{pmatrix} \begin{pmatrix} D_{ee}^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -K(M_4)_{eo}M_5^{-1} \\ 0 & 1 \end{pmatrix}$$

$$D_{ee} = M_5 - K^2(M_4)_{eo}M_5^{-1}(M_4)_{oe}$$

- Polynomial accelerated
  P_n(H_DWF)
- With shift
  H-> H-c
- eigen Compression / decompression

$$\psi = v_1 + v_2$$
$$H(\psi) = \lambda_1 v_1 + \lambda_2 v_2$$



Low-mode decomposition

- 4D even-odd decomposition

$$D_{DW} = \begin{pmatrix} M_{5\,ee} & KM_{4\,eo} \\ KM_{4\,oe} & M_{5\,oo} \end{pmatrix}$$

M_5 : with 5D differential, 4D diagonal
M_4 : with 4D differential, 5D diagonal

$$= \begin{pmatrix} 1 & KM_{4\,eo}M_{5\,oo}^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D_{ee} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ KM_{4\,oe} & M_{5\,oo} \end{pmatrix}$$

$$D_{ee} = M_5 - K^2 M_{4\,eo} M_{5\,oo}^{-1} M_{4\,oe}$$

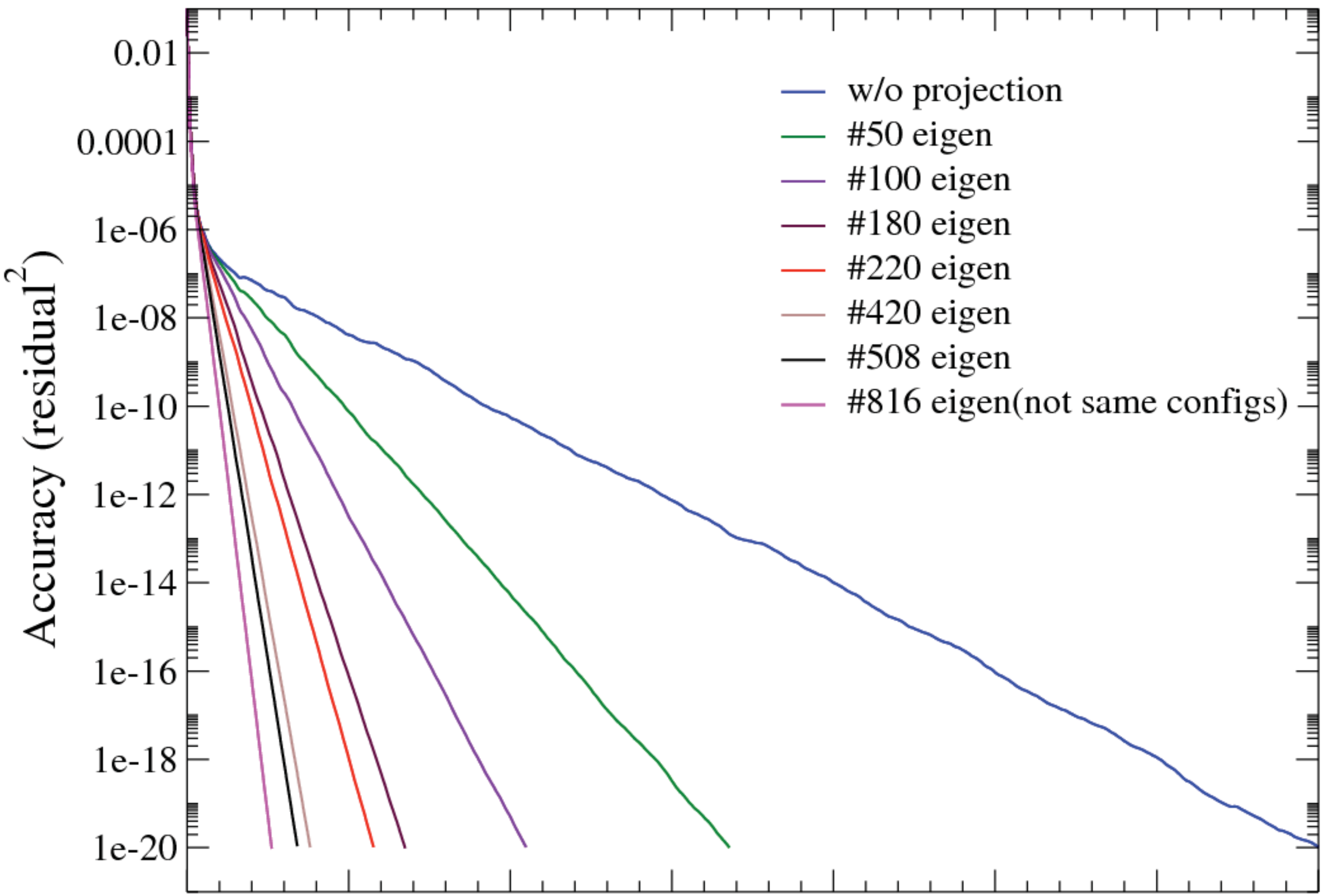
$$D_{DW}^{-1} = \begin{pmatrix} 1 & 0 \\ -KM_{5\,oo}^{-1}M_{4\,oe} & M_{5\,oo}^{-1} \end{pmatrix} \begin{pmatrix} D_{ee}^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -KM_{4\,eo}M_{5\,oo}^{-1} \\ 0 & 1 \end{pmatrix}$$

- Low mode decomposition

$$D_{ee}^{-1} = D_{\text{low}\,ee}^{-1} + D_{\text{high}\,ee}^{-1}$$

$$D_{\text{low}\,ee}^{-1} = H_{\text{low}\,ee}^{-2} D_{ee}^{\dagger} = \sum_k \frac{1}{\lambda_k^2} \psi_k (D_{ee}\psi_k)^{\dagger}, \quad H_{ee}\psi_k = \lambda_k \psi_k, \quad H_{ee} = \Gamma_5 D_{ee}$$

$$D_{\text{low}\,DW}^{-1} = \begin{pmatrix} 1 & 0 \\ -KM_{5\,oo}^{-1}M_{4\,oe} & M_{5\,oo}^{-1} \end{pmatrix} \begin{pmatrix} D_{\text{low}\,ee}^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -KM_{4\,eo}M_{5\,oo}^{-1} \\ 0 & 1 \end{pmatrix}$$


Accuracy (residual^2)
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14
1e-16
1e-18
1e-20
w/o projection
#50 eigen
#100 eigen
#180 eigen
#220 eigen
#420 eigen
#508 eigen
#816 eigen(not same configs)

Examples of Covariant Approximations (contd.)

- **All Mode Averaging AMA**

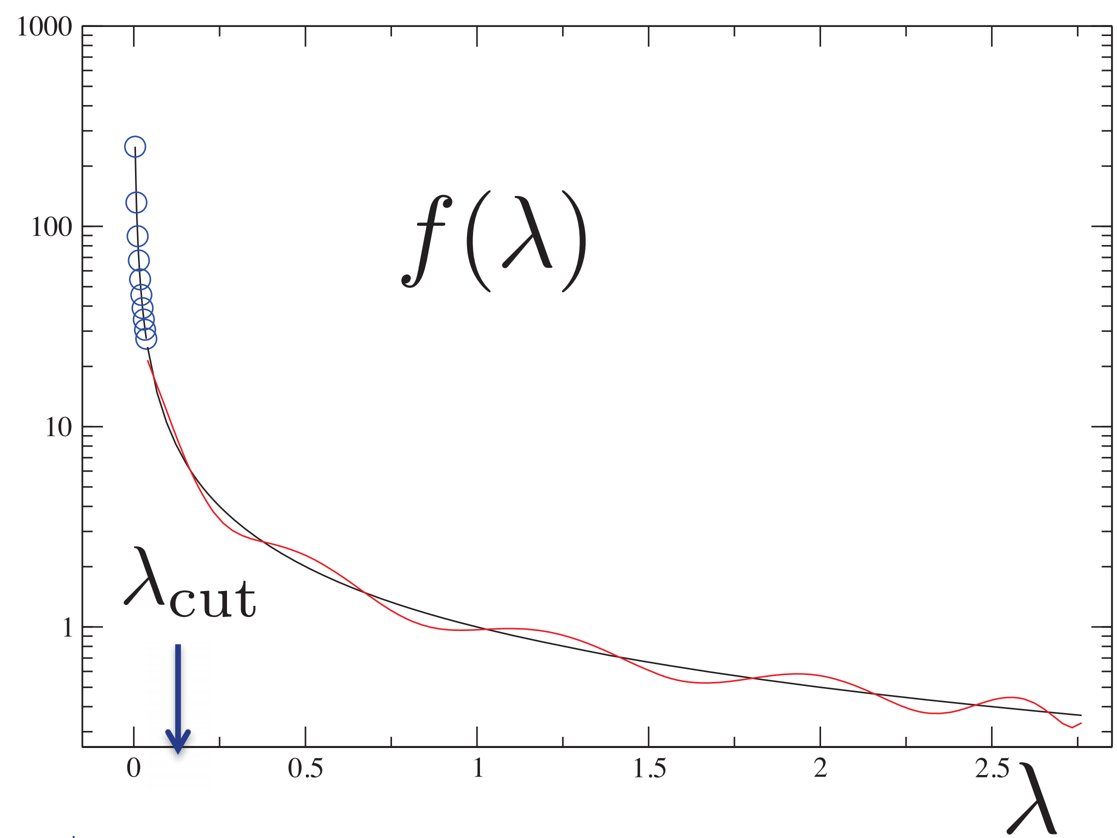
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

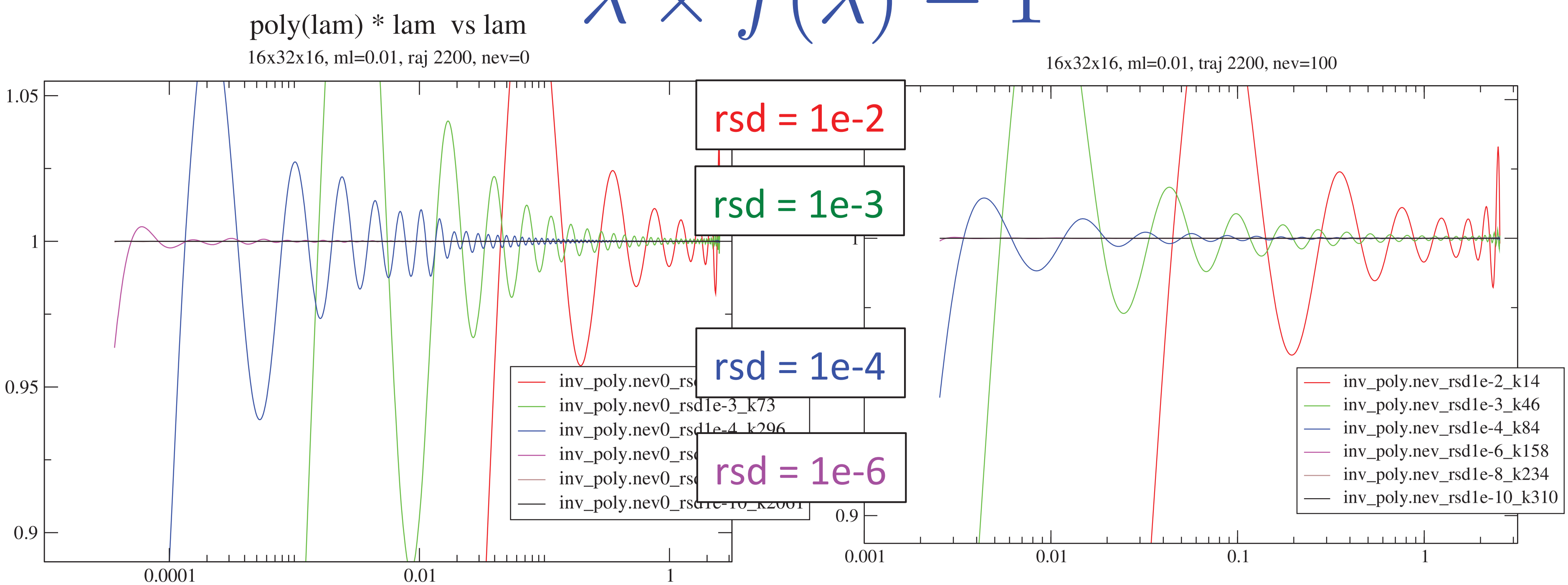
$$P_n(\lambda) \approx \frac{1}{\lambda}$$



accuracy control :
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

All mode approximation via sloppy CG

$$\lambda \times f(\lambda) - 1$$



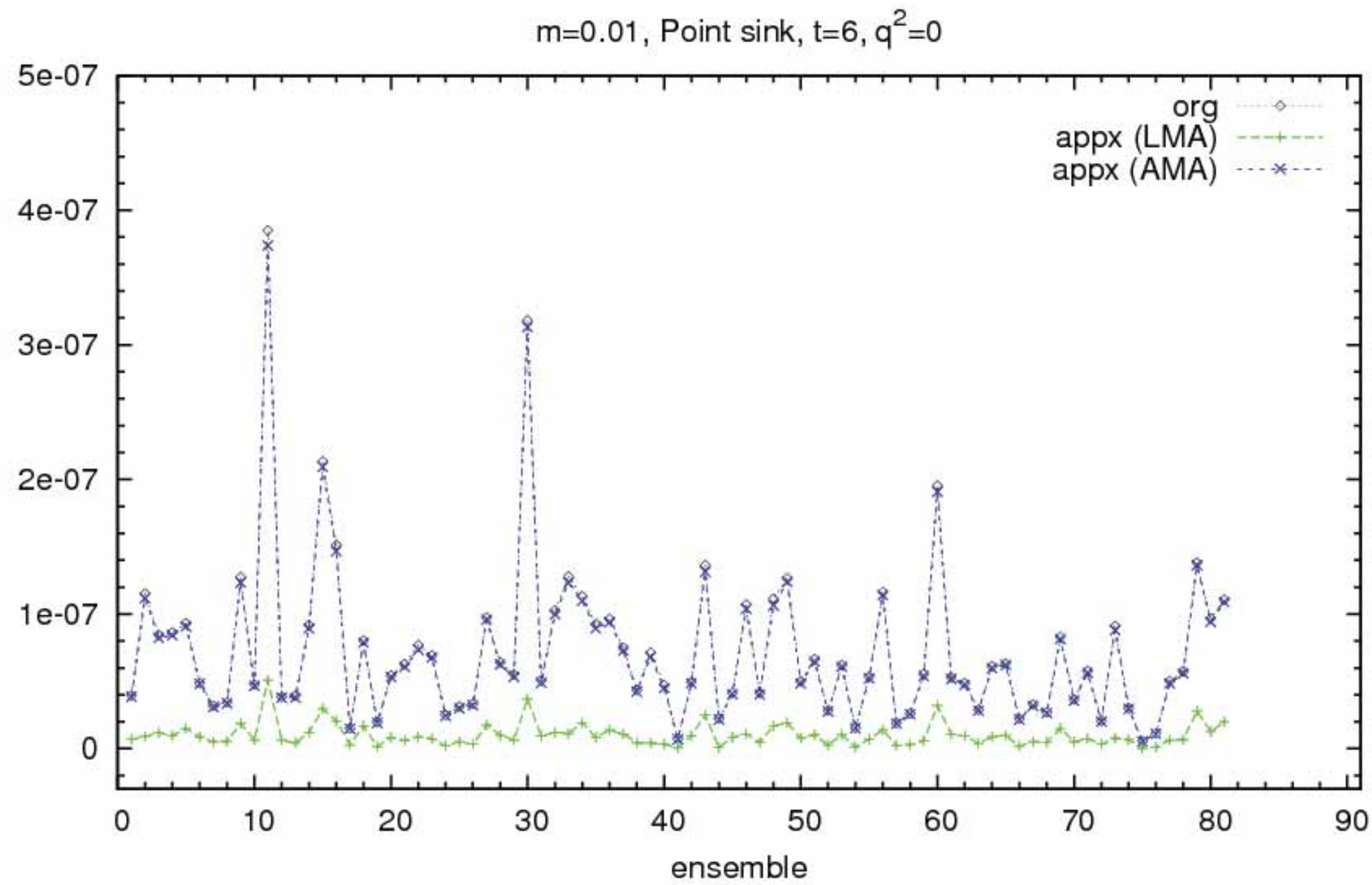
no eigenvector assists

100 eigenvector assists

- Conjugate residual with sloppy convergence criteria, which is equivalent to construct a polynomial approximating 1/λ
- The starting vector needs to be <u>translation invariant</u> to be a covariant approx.
- low eigenvectors reduces the size of the dynamic range of 1/λ
  → Better approximation with smaller polynomial degrees
- low λ region has larger relative errors
- One could employ other construction of polynomial approximation for 1/λ, such as min-max, conjugate rasidual

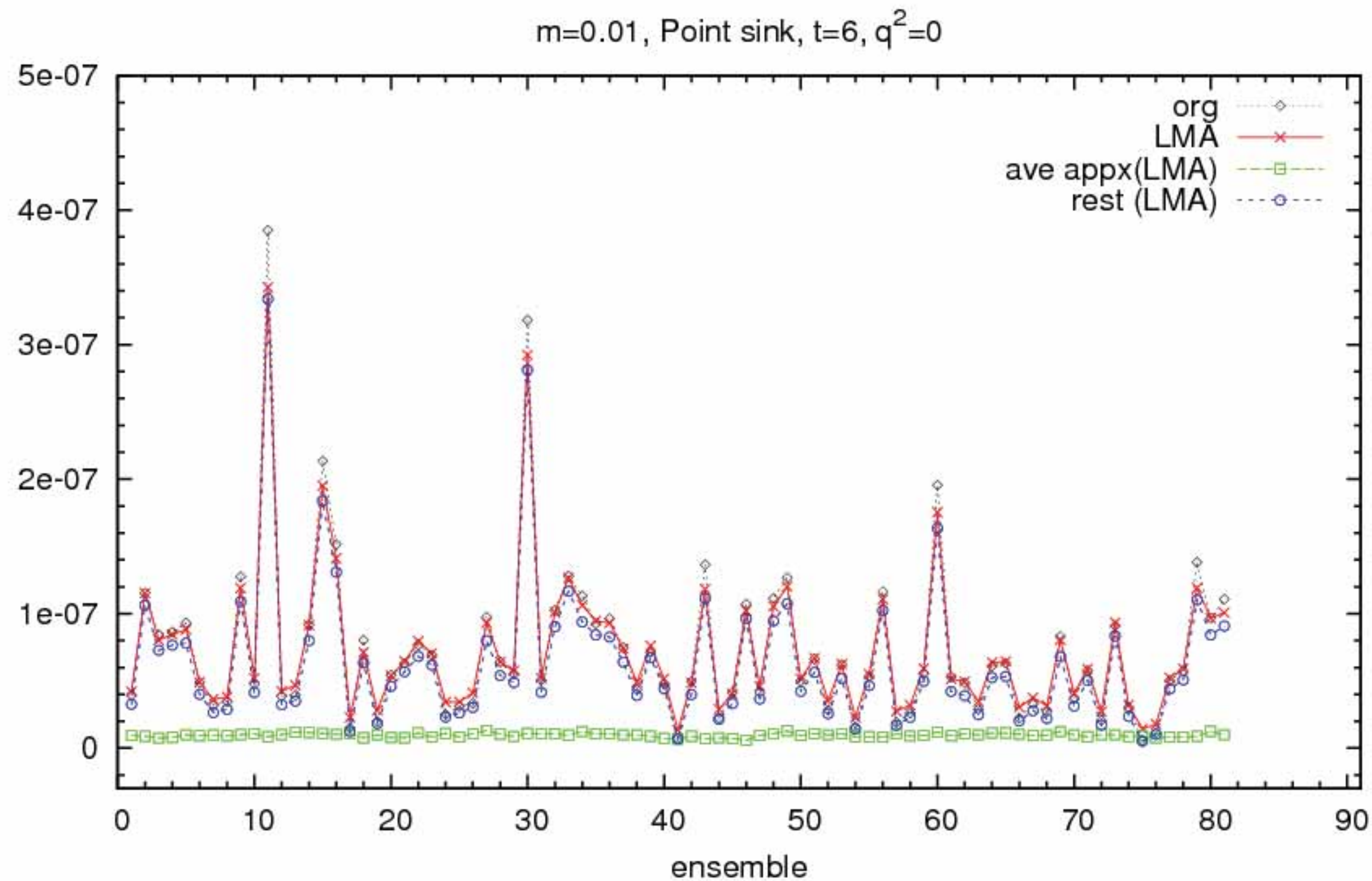
Correlation

- NN propagator at short time-slice



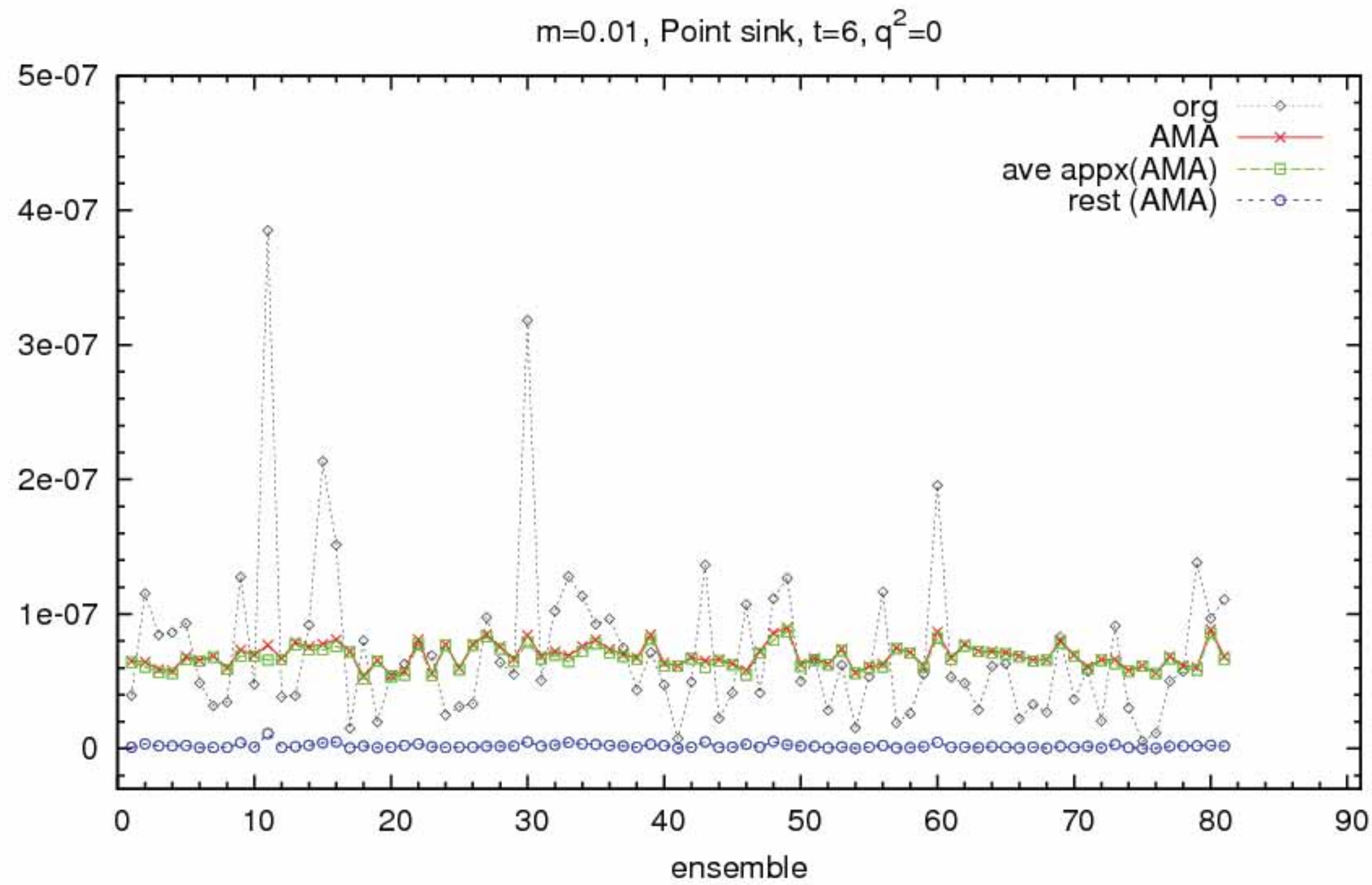
Correlation

- NN propagator (LMA) at short time-slice



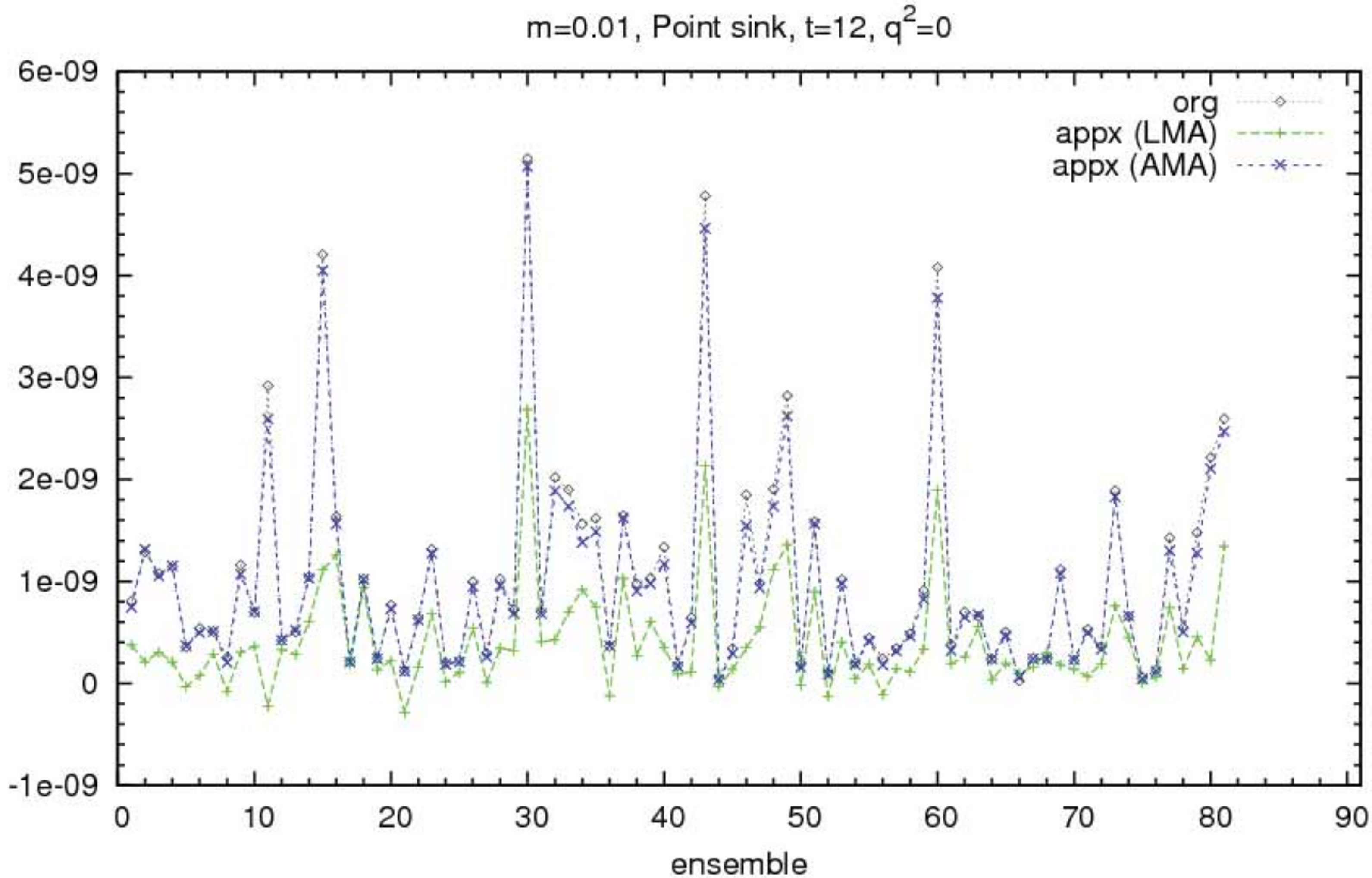
Correlation

- NN propagator (AMA) at short time-slice



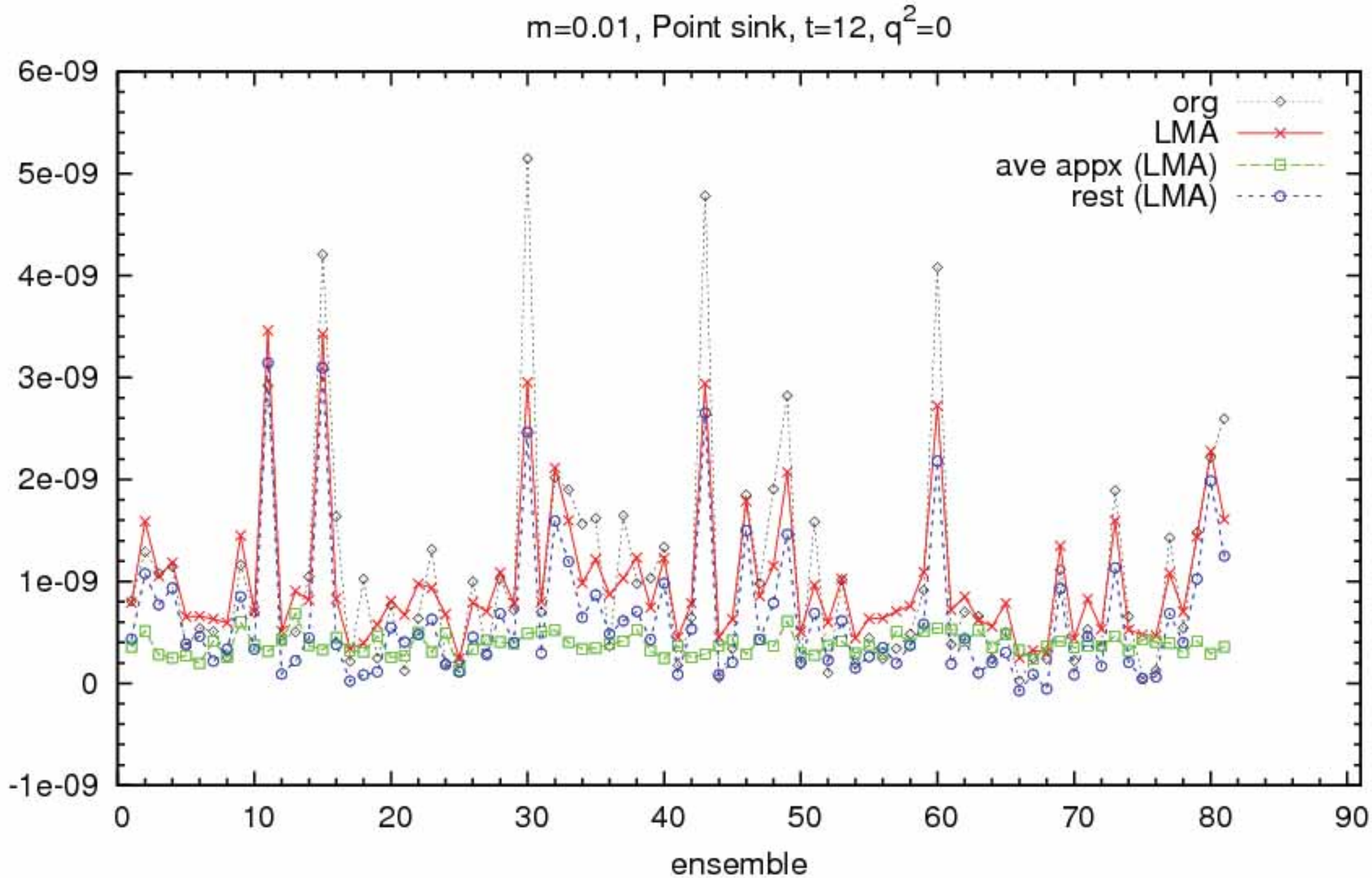
Correlation

- NN propagator at long time-slice



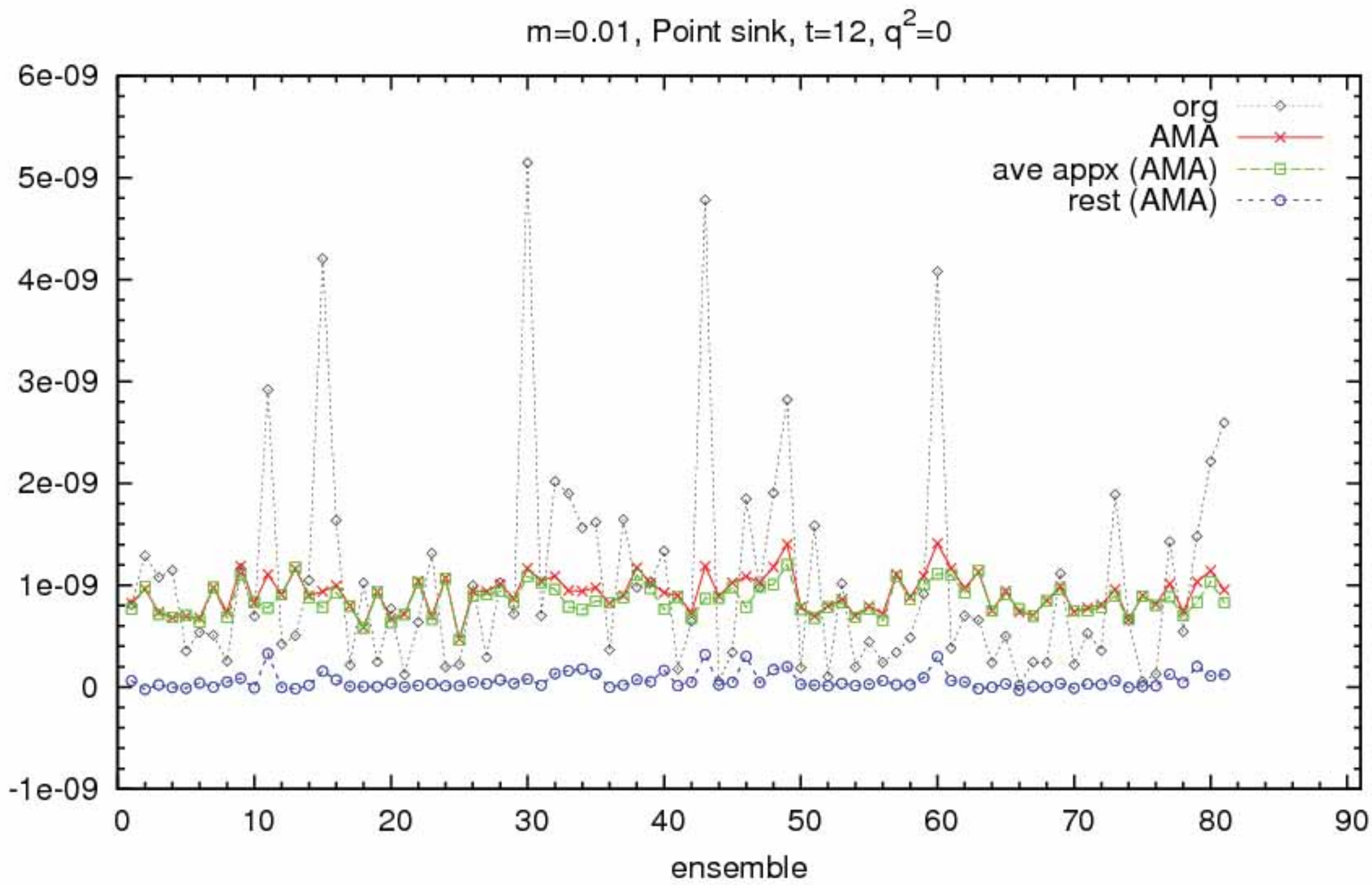
Correlation

- NN propagator (LMA) at long time-slice

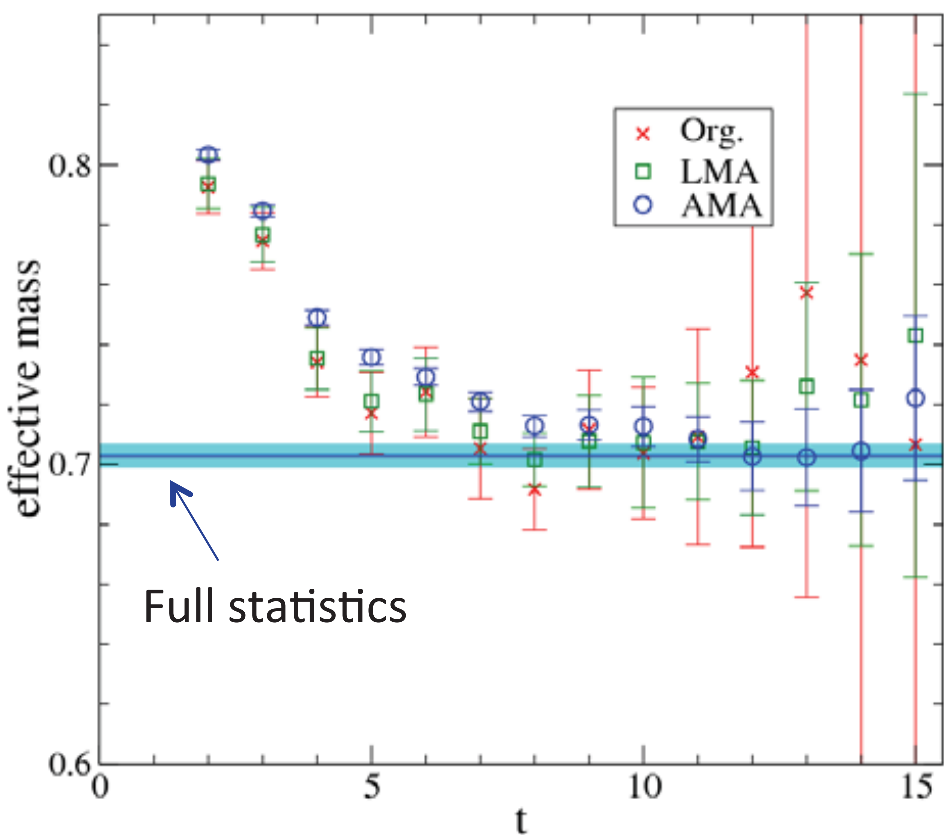


Correlation

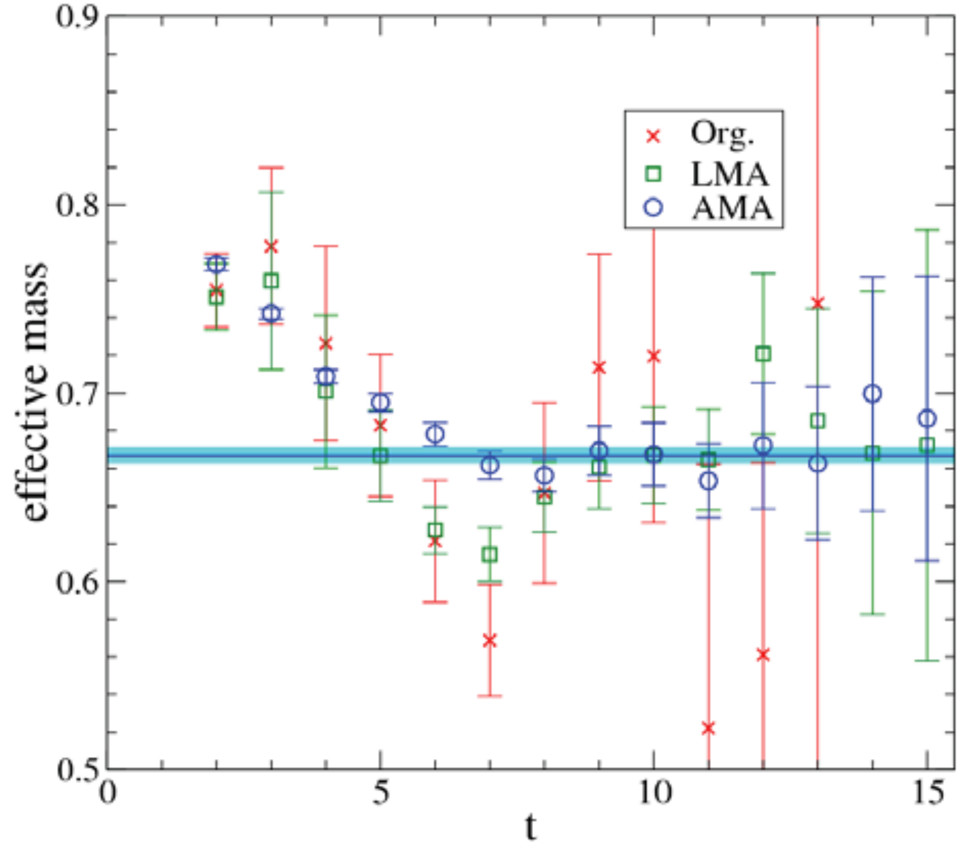
- NN propagator (AMA) at long time-slice



N, m=0.01, point sink



N, m=0.005, point sink



	LMA [7,15]	AMA [7,15]	statistics	Full statistics (Gaussian sink)
m = 0.01	0.712(16)	0.710(5)	N_{conf}=80, N'_{mes}=32	0.703(4), N_{conf} = 356, N_{mes}=4
m = 0.005	0.673(22)	0.666(13)	N_{conf}=26, N'_{mes}=32	0.663(4), N_{conf} = 932, N_{mes}=4

Yamazaki et al., PRD79, 114505 (2009)

Cost (in the case of 24cube m=0.01)

Use of unit of quark propagator "prop" in full CG w/o deflation

Yamazaki et al., PRD79, 114505 (2009)

- Case of full statistics

  In N_{conf} = 356, N_{mes}=4,

  Total : 356×4 = 1424 prop

- Case of AMA w/o deflation

  Since calculation of O^{appx} need 1/50 prop, then in N_{conf}=81, N'_{mes}=32

  Total : 80 + 80×32/50 = 131 prop ⇒ 10 times fast

- Case of AMA w/ deflation

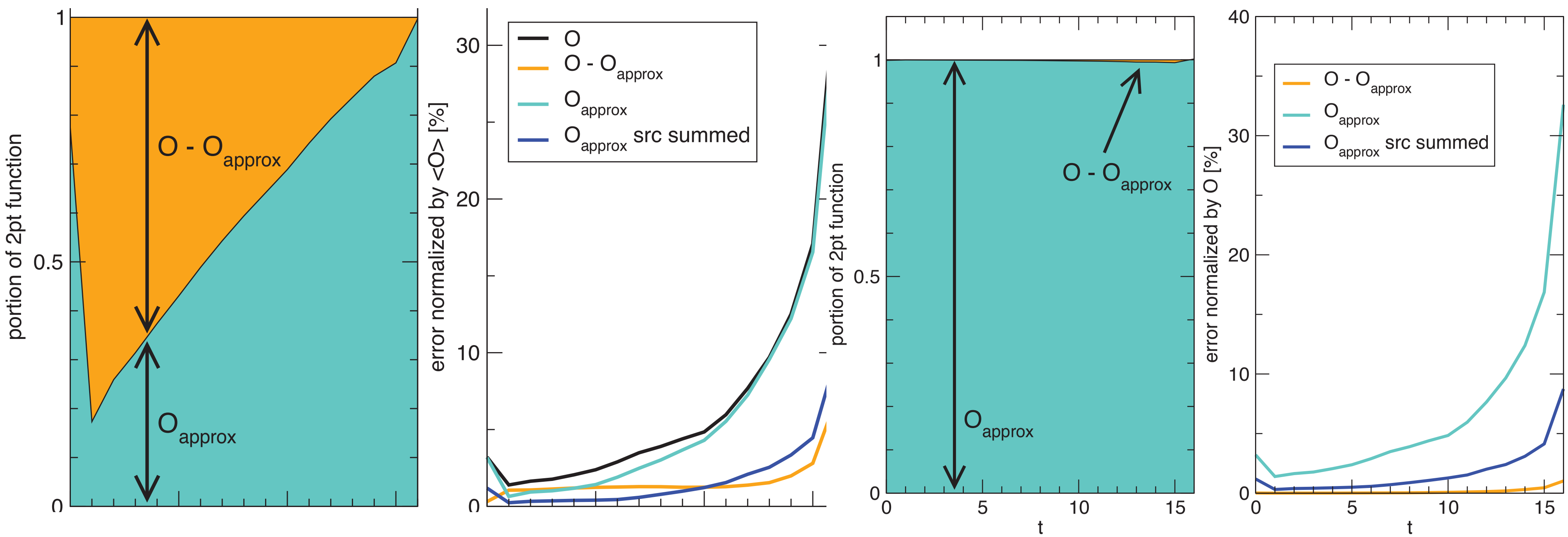
  When using 180 eigenmode, calculation of O^{appx} need 1/80 prop, but in this case the calculation of lowmode is ~1 prop/configs. Deflated CG makes reduction of full CG to 1/3 prop, then

  Total : 80/3 + 80×32/80 + 80 = 138 prop ⇒ 10 times fast

  Note that stored eigehmode is useful for other works.

AMA in USQCD Static-light
[PI Tomomi Ishikawa]

16^3x64x16, 20 conf, 100 eigenvectors



LMA

AMA

3pt function [E. Shintani]

- Application to the form factor measurement
  - CP-even and CP-odd nucleon EM form factor

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta\Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\rm P,T\text{-}odd} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\rm P,T\text{-}even} + \cdots\Big]u_N^\theta$$

  - Complicated structure in the ratio method

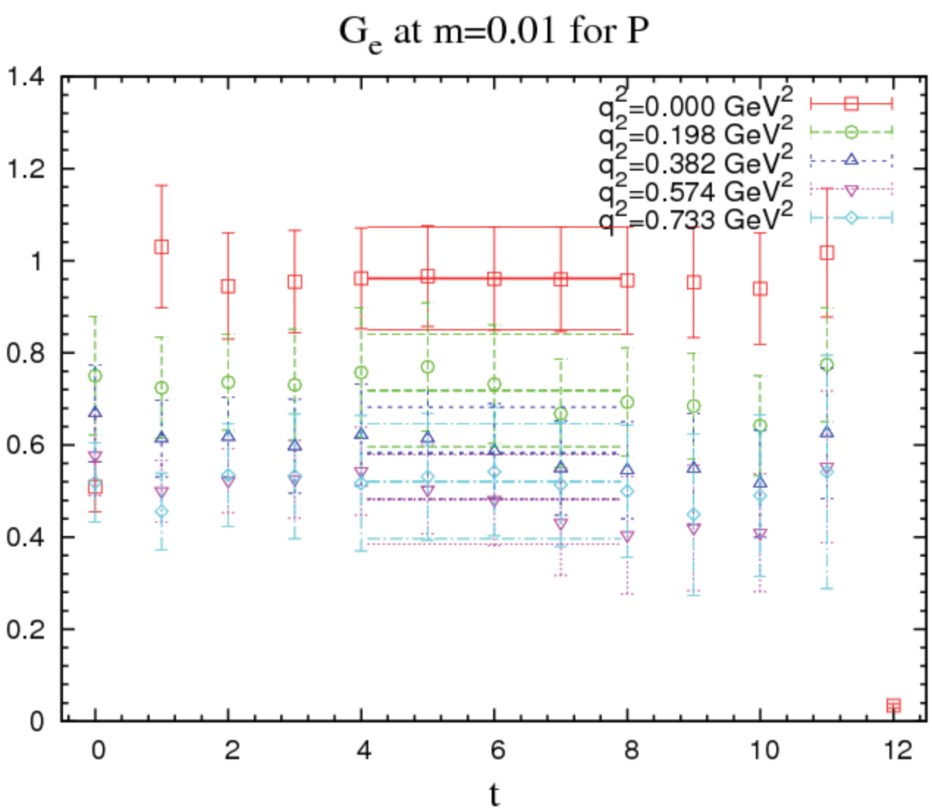
Cf. Yamazaki et al., PRD79, 114505 (2009)

$$R_{J_\mu}(t,\vec{q}) = \sqrt{\frac{m_N}{2(E_N+m_N)}}\frac{\langle\eta_N^g J_\mu\bar{\eta}_N^g\rangle(t,\vec{q})}{\langle\eta_N^l\bar{\eta}_N^g\rangle(t_{\rm snk}-t_{\rm src},0)}R(t,\vec{q}),$$

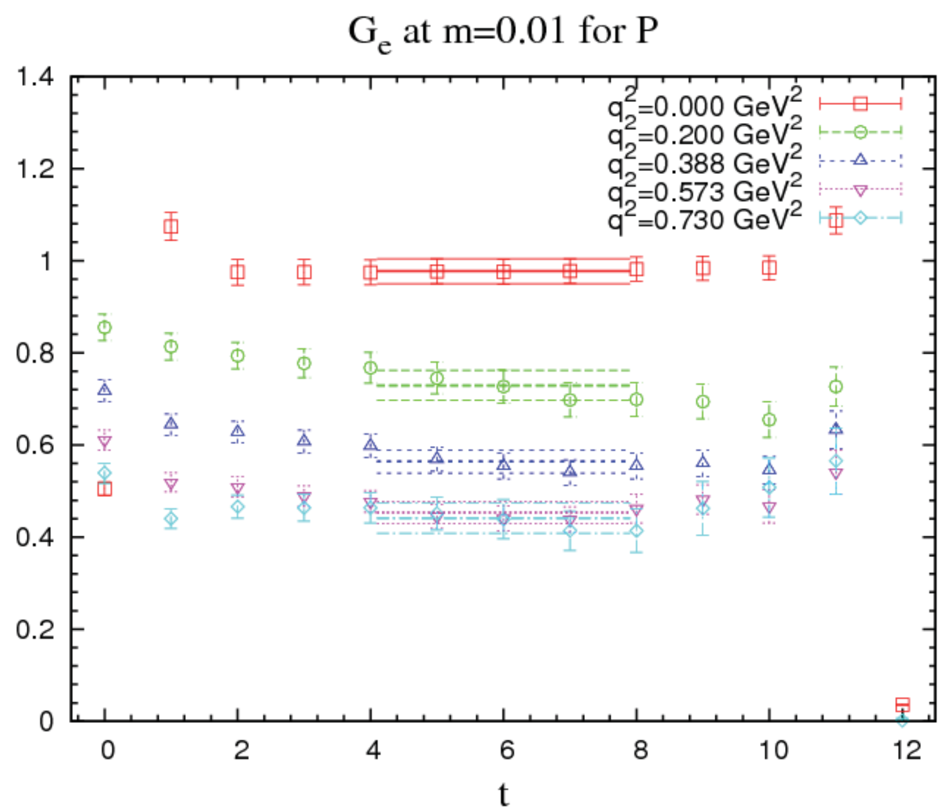
$$R(t,\vec{q}) = \left[\frac{\langle\eta_N^l\bar{\eta}_N^g\rangle(t_{\rm snk}-t,\vec{q})\langle\eta_N^g\bar{\eta}_N^g\rangle(t-t_{\rm src},0)\langle\eta_N^l\bar{\eta}_N^g\rangle(t_{\rm snk}-t_{\rm src},0)}{\langle\eta_N^l\bar{\eta}_N^g\rangle(t_{\rm snk}-t,0)\langle\eta_N^g\bar{\eta}_N^g\rangle(t-t_{\rm src},\vec{q})\langle\eta_N^l\bar{\eta}_N^g\rangle(t_{\rm snk}-t_{\rm src},\vec{q})}\right]^{1/2}$$

Ratio has complicated combination of both low and high mode,
so AMA has more advantage than LMA even if AMA need larger cost.

LMA



AMA



q^2 GeV^2	G_e (LMA)	G_e (AMA)
0.0	0.96(11)	0.98(3)
0.198	0.72(12)	0.73(3)
0.382	0.58(10)	0.56(3)
0.574	0.48(10)	0.45(2)
0.733	0.52(12)	0.44(3)

Statistical error of AMA is about 3--5 times smaller than LMA.

CP-odd part

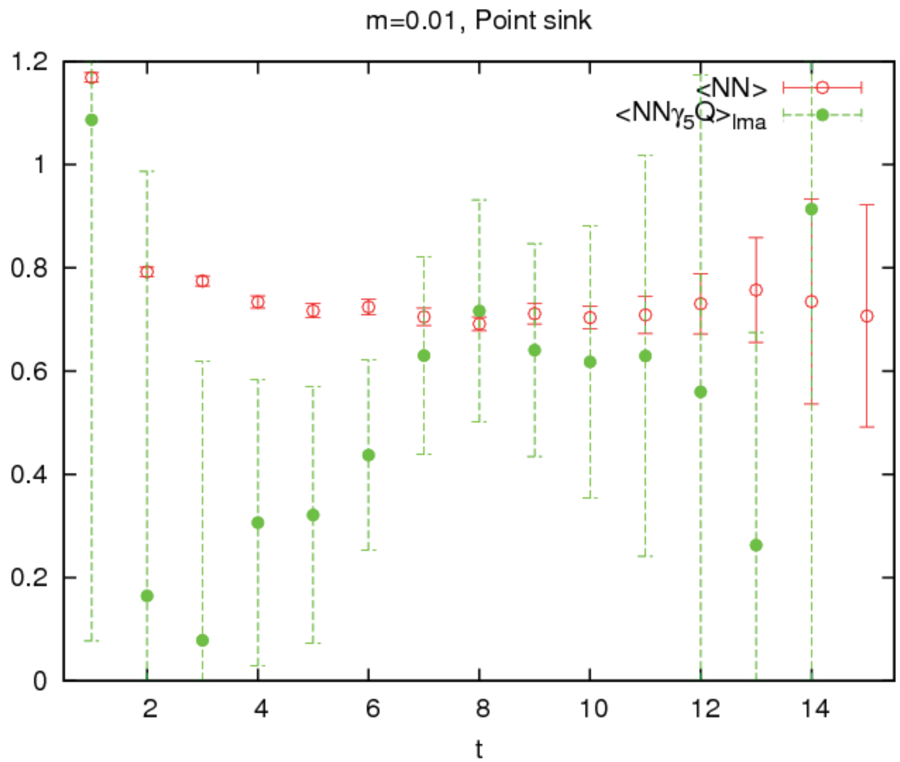
- Nucleon 2pt function with θ reweighting

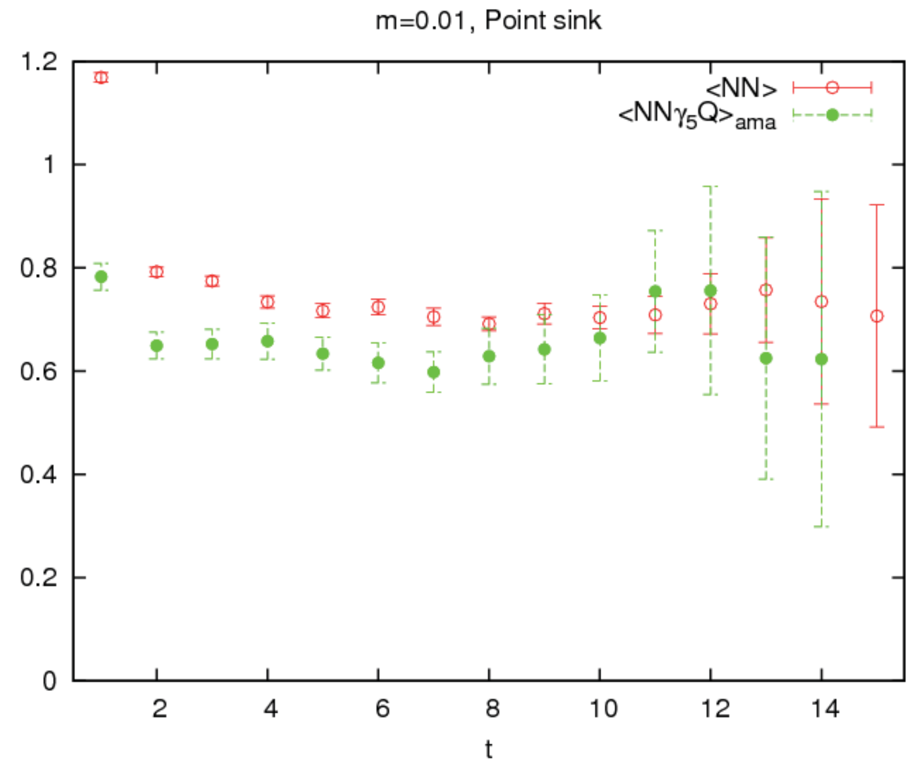
$$\langle \eta_N \bar{\eta}_N \rangle_\theta(\vec{p}) = Z_N^2 \frac{ip\cdot\gamma + m_N e^{i\alpha(\theta)\gamma_5}}{2E_N}$$

$$\mathrm{tr}\left[\gamma_5 \langle Q \eta_N \bar{\eta}_N \rangle(\vec{p})\right] \simeq Z_N^2 \frac{2m_N}{E_N} \alpha e^{-E_N t}$$

  - Q is topological charge.
  - α which is CP-odd phase is necessary to extract EDM form factor.
  - It is good check of applicability of LMA/AMA to CP-odd sector.
  - Effective mass plot shows the consistency of the above formula

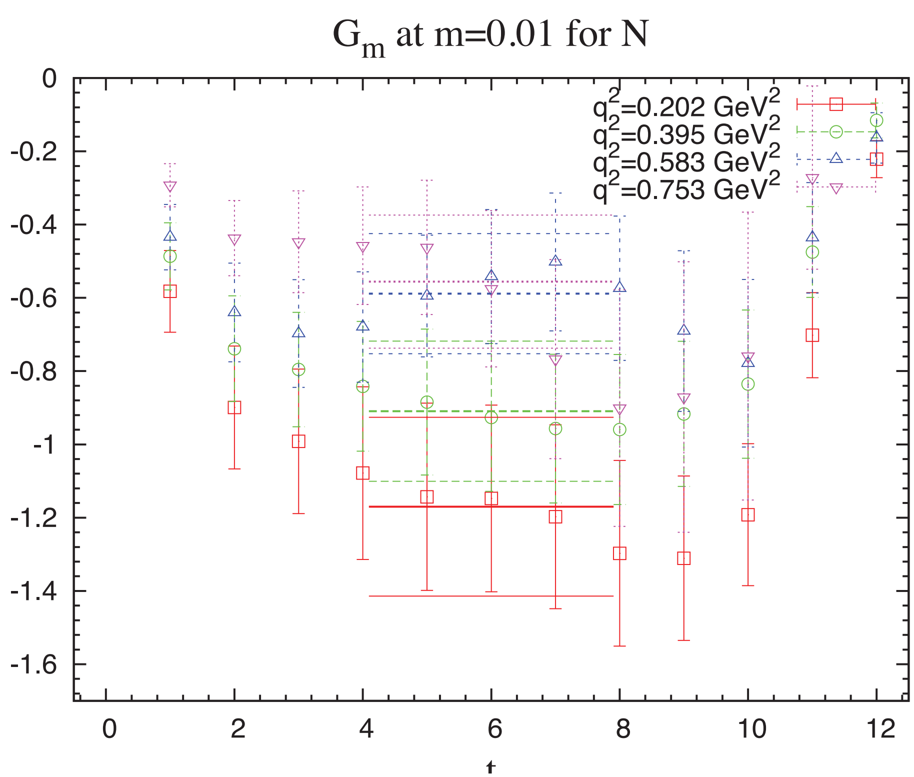
CP-odd part [E. Shintani]



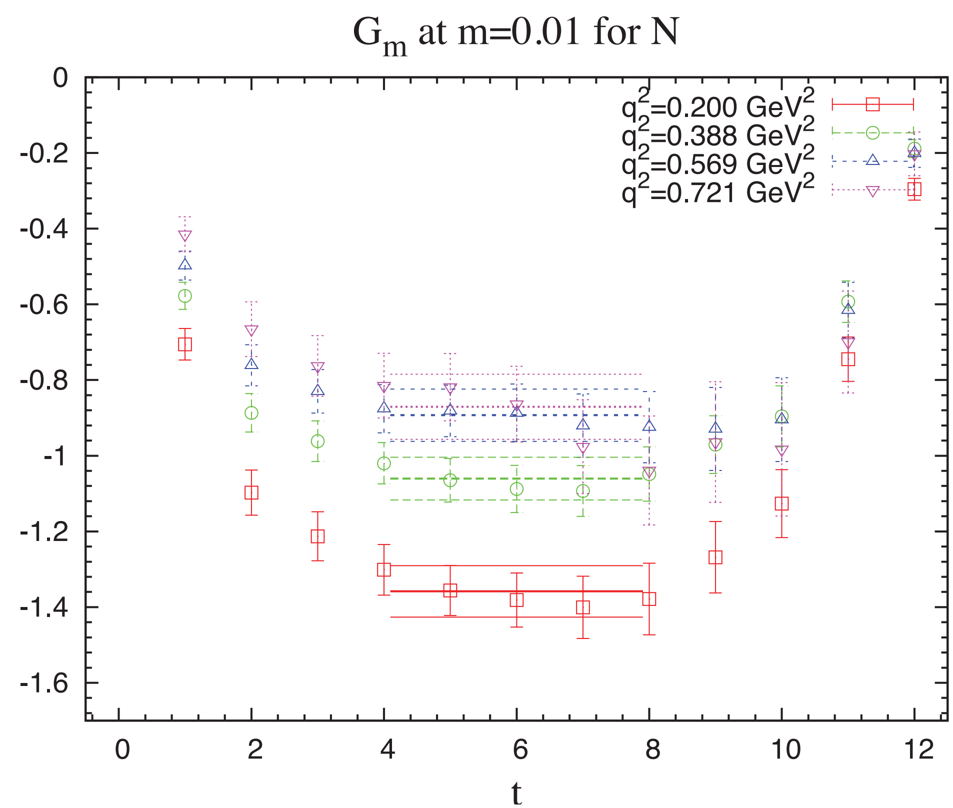


- There is good plateau in AMA, and this figure actually shows CP-odd part has consistent exponent with CP-even(nucleon mass) part as expected.
- CP-odd part has both contribution from high and low lying mode.
- AMA works well even in CP-odd sector !

Nucleon Magnetic formfactor



Original CG



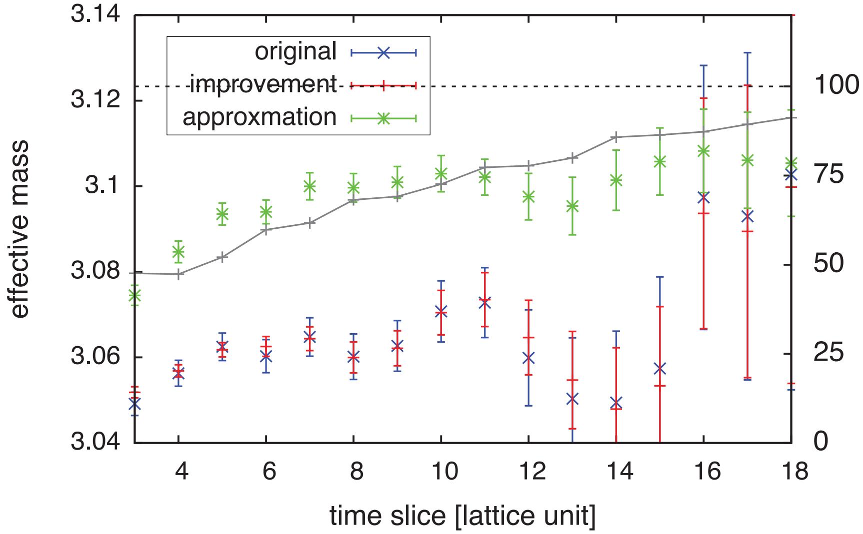
AMA

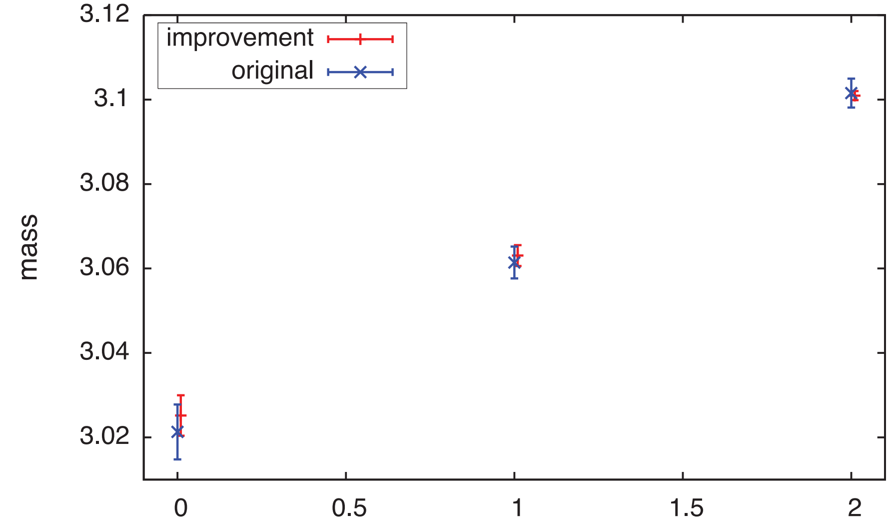
Variants of CAA

- CAA (Covariant Approximation Averaging)
  - Name
    approximation,
    approximation accuracy control
  - LMA (Low Mode Averaging)
    low mode approx of propagator,
    # of eigen vectors
  - AMA (All Mode Averaging),
    low mode (optional)+Polynomial approx,
    (# of eigenV) Polynomial degree
    (also other type of minimization)
  - Heavy quark averaging [T. Kawanai]
    heavier mass quark prop as an approx of light prop
    quark mass
  - ?????

Larger mass as CAA
[Taichi Kawanai]

24^3x64x16, 20 config ,
mf=0.01 (target) mf=0.04 “approximation”





Other Examples of Covariant Approximations

- Less expensive (parameters of) fermions :
  - Larger mf
  - Smaller Ls DWF
  - Mobius
  - even staggered or Wilson .....
- Different boundary conditions
- More than one kinds of approximation (c.f. multi mass Hasenbushing)

Strongly depends on Observables / Physics (YMMV)

Would work better for EXPENSIVE observables and/or fermion, potentially a game changer ?

Other related/ similar techniques

- LMA

  L. Giusti, P. Hernandez, M. Laine, P. Weisz and H. Wittig, JHEP 0404, 013 (2004)
  see also H. Neff, N. Eicker, T. Lippert, J. W. Negele and K. Schilling, Phys. Rev. D 64 (2001) 114509 and T. DeGrand and S. Schaefer, Comput. Phys. Commun. 159 (2004) 185

  works for low mode dominant quantities

- Truncated Solver Method (TSM)

  G. Bali, S. Collins, A. Schaefer, Comput. Phys. Commun. 181 (2010) 1570

  uses stochastic noise to avoid systematic error

- All-to-all propagator

  J.Foley, K.Juge, A. O'Cais, M. Peardon, S. Ryan, J-I. Skullerud, Comput.Phys.Commun. 172 (2005) 145

  uses stochastic noise
  could use CAA as a part of A2A

Summary

- CAA , LMA, AMA, ... : Class of Statistical error reduction technique

  - AMA is a valence version of the Hasenbush trick
  - AMA could improve existing data easily

1. Do Full CG for selected config / source
   (existing data : This expensive part is already done)
2. Find a good approximation (accuracy of sloppiness / number of eigenvalue) that reproduce your exact CG result by, say, 95%
   (mathematically find a strongly correlated approximation, R(corr) > 0.5)
3. Subtract the approx obs with same source location as full CG

$$\mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

4. Perform many source location using approx obs, average, add back

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

You could use other config.

- YMMV, find a good / cheap / funny approximations

Other technical details

- Implicitly Restarted Lanczos with Polynomial acceleration and spectrum shifts for DWF and staggered in CPS++ [E. Shintani, T. Blum, TI].
- Eigen Vector compression / decompression
- Sea Electric Charge is now controlled by QED reweighting
  [T. Ishikawa et. al. arXiv:1202.6018]
- Aslash-SeqSrc method