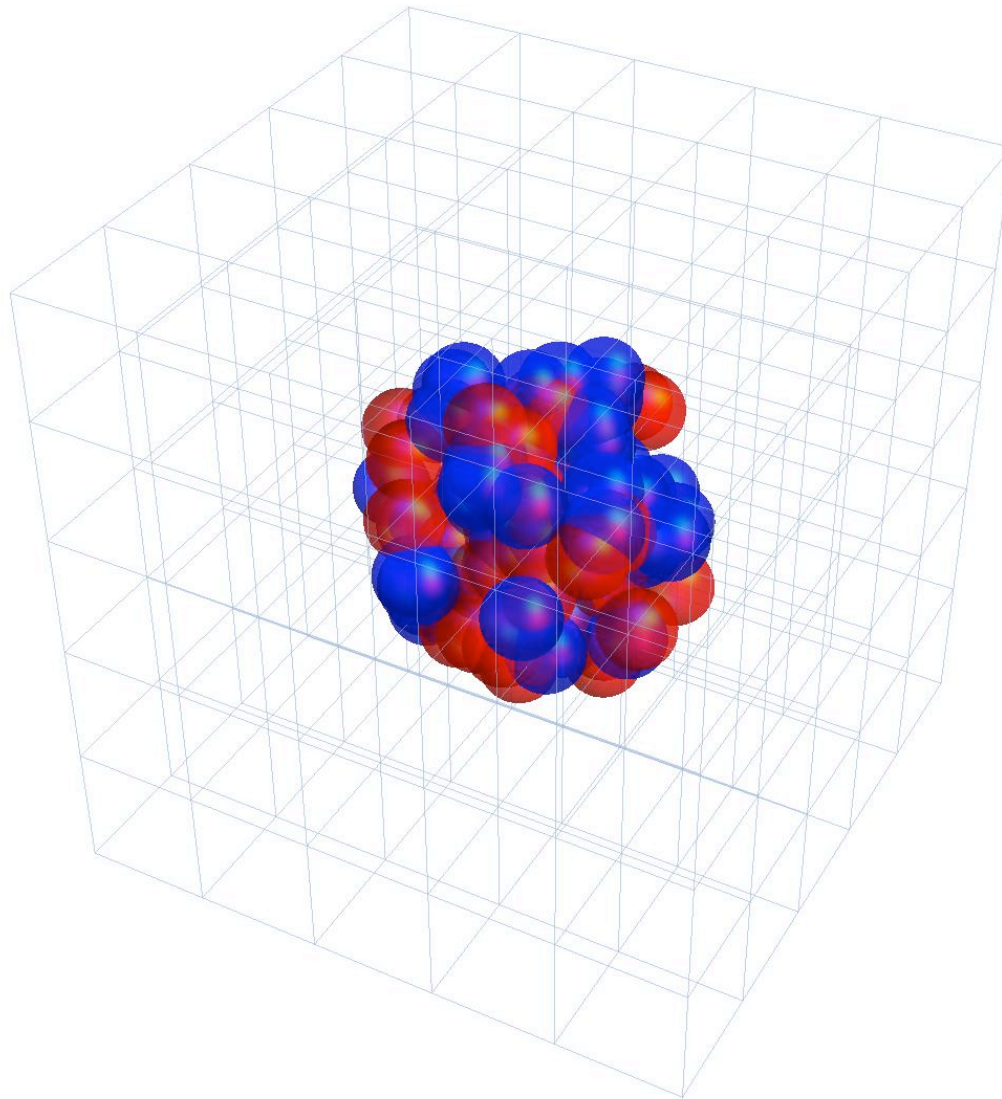


Nuclei and Hadronic Interactions in LQCD

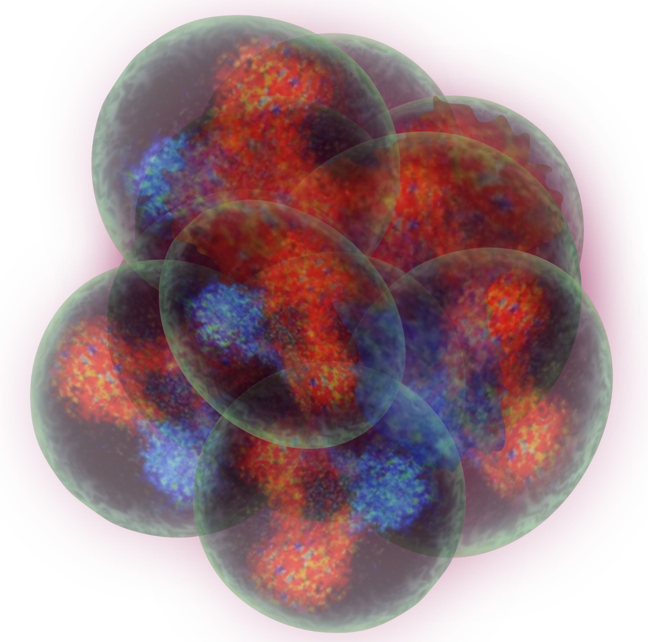
William Detmold

Massachusetts Institute of Technology

Massachusetts Institute of Technology

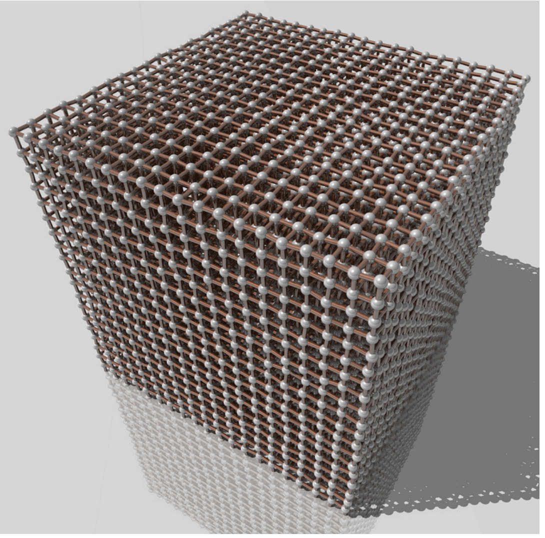
From quarks to nuclei

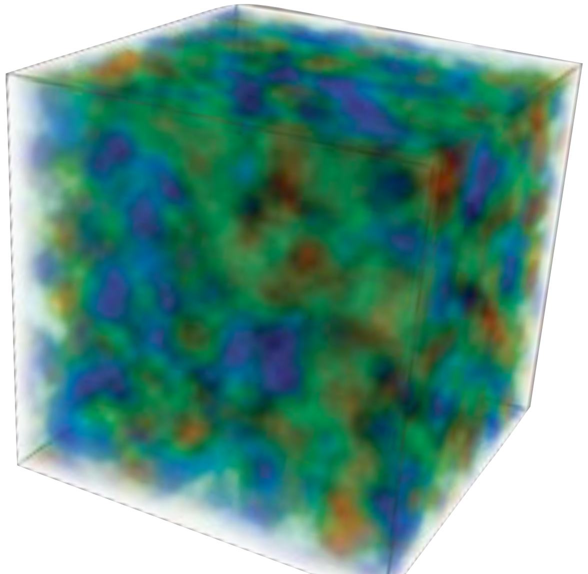
- Nuclear physics: an emergent phenomenon of the Standard Model
  - Like the proton, nuclei are SM eigenstates
- How *exactly* do nuclei emerge from the SM?
  - Issues
  - Recent progress
  - Future prospects



Quantum chromodynamics

- NP from the SM, but focus on QCD
- Lattice QCD: quarks and gluons
  - Formulate problem as functional integral over quark and gluon d.o.f. on R_4
  - Discretise and compactify system
  - Integrate via importance sampling (average over "important" cfgs)
  - *Systematically* undo the harm done in previous steps

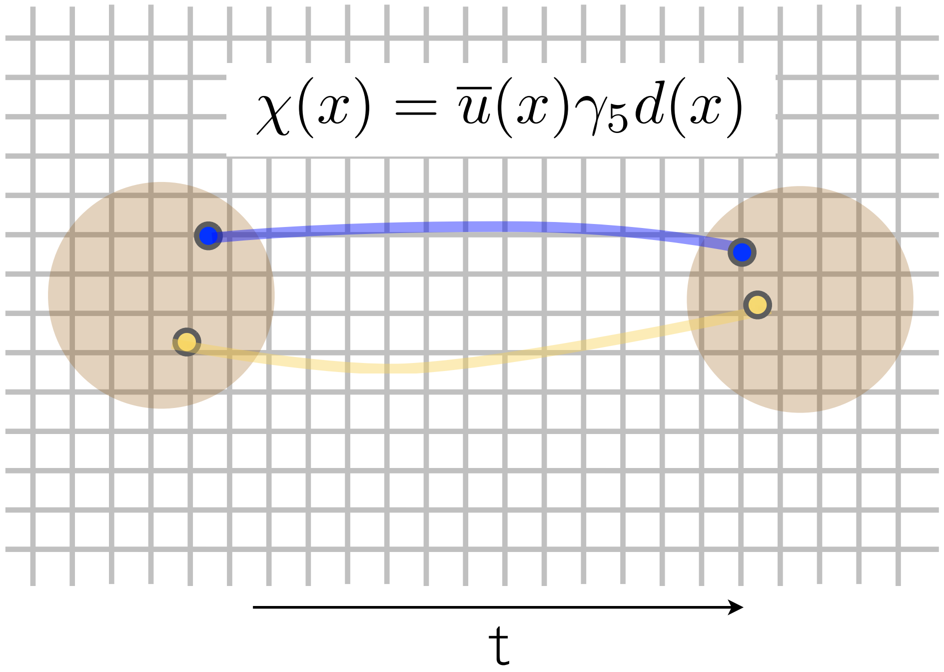




QCD Spectroscopy

- Measure correlator (χ = object with q# of hadron)

$$C_2(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(\mathbf{0},0)|0\rangle$$



- Unitarity: $\sum_n |n\rangle\langle n| = 1$

$$= \sum_{\mathbf{x}} \sum_n \langle 0|\chi(\mathbf{x},t)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Hamiltonian evolution

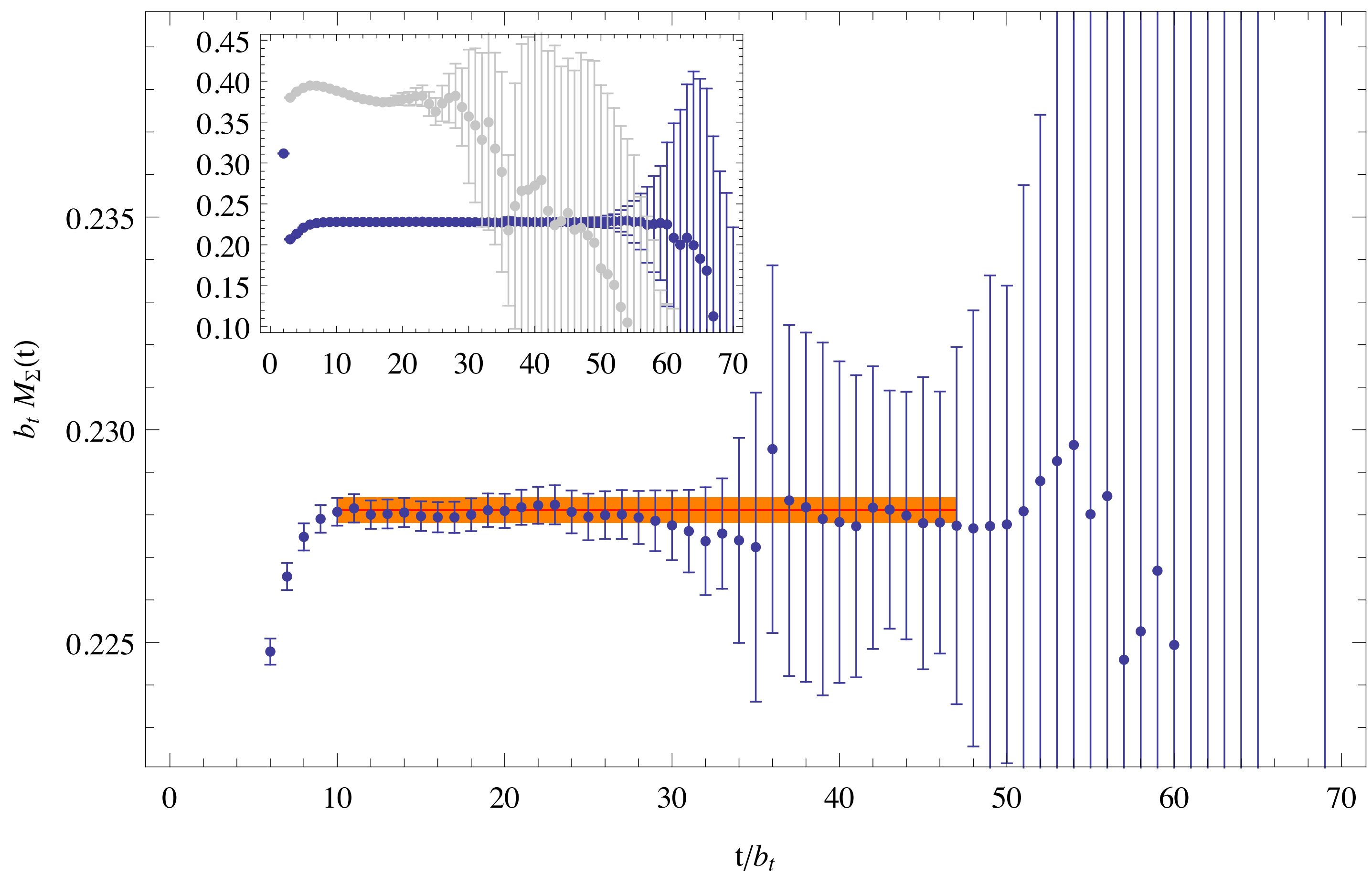
$$= \sum_{\mathbf{x}} \sum_n e^{-E_n t} e^{i\mathbf{p}_n\cdot\mathbf{x}} \langle 0|\chi(\mathbf{0},0)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Long times only ground state survives

$$\xrightarrow{t\to\infty} e^{-E_0(\mathbf{0})t} |\langle \mathbf{0};0|\overline{\chi}(\mathbf{x_0},t)|0\rangle|^2 = Z\, e^{-E_0(\mathbf{0})t}$$

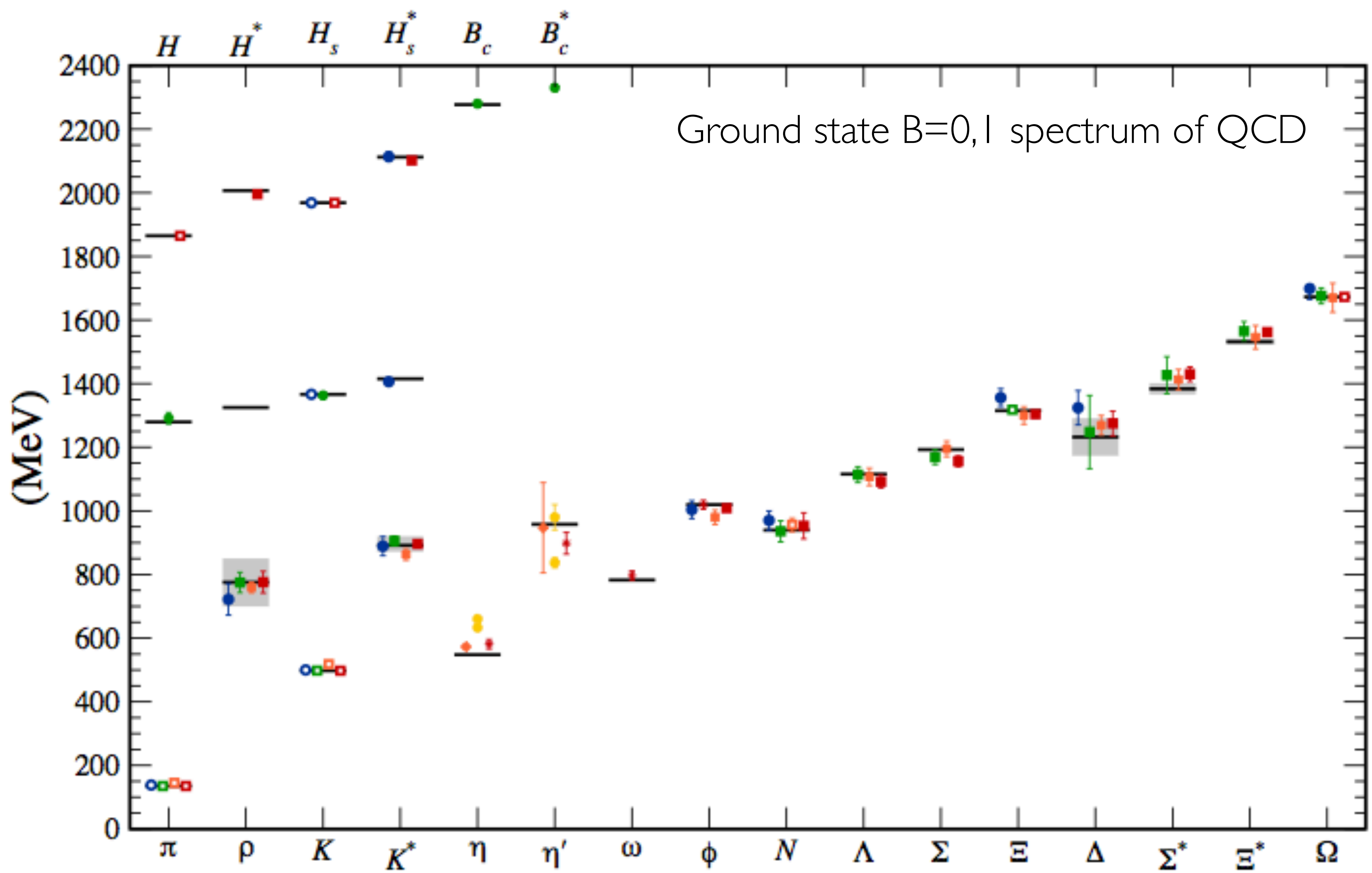
Effective mass

- Construct $M(t) = \ln\left[C_2(t)/C_2(t+1)\right] \stackrel{t\to\infty}{\longrightarrow} M$
  - Plateau corresponds to energy of ground state



- Fancier techniques able to resolve multiple eigenstates

QCD: meson/baryon spectrum



[recent review: A Kronfeld, 1209.3468]

points correspond to calculations by different groups

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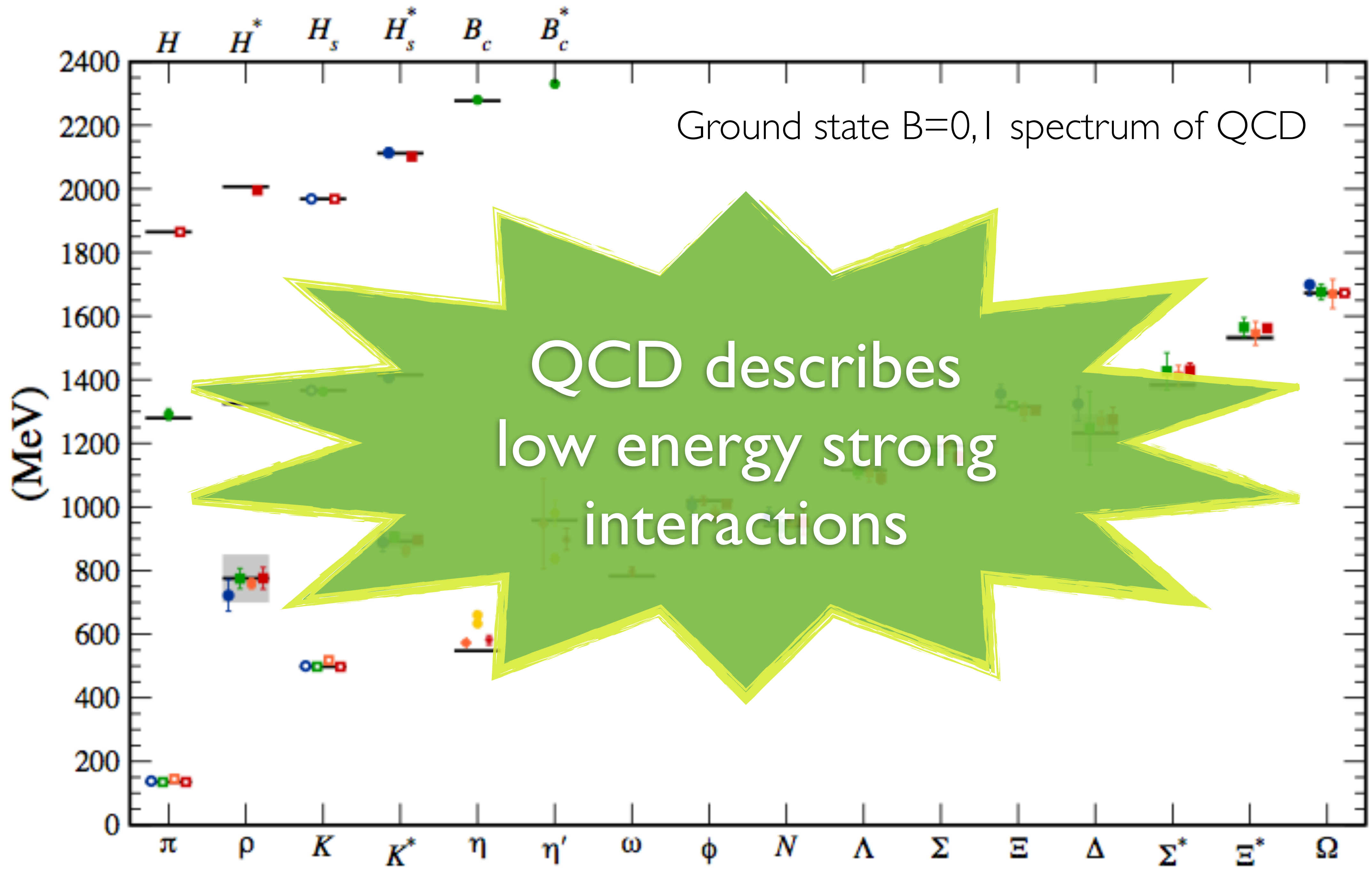

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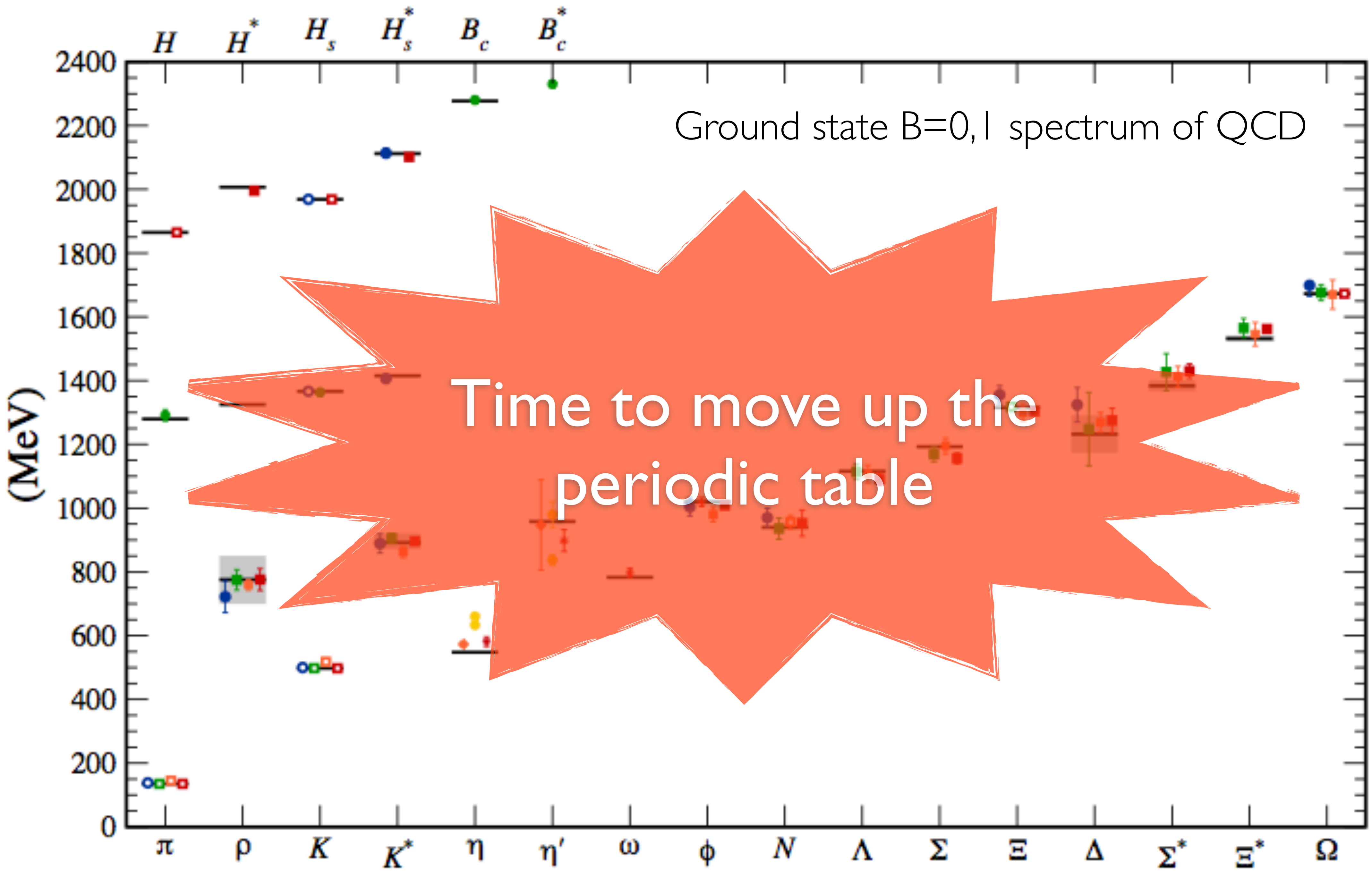

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Nuclear physics from LQCD

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- Can we compute the mass of ^{208}Pb in QCD?

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$$\langle 0 | T q_1(t) \dots q_{624}(t) \overline{q}_1(0) \dots \overline{q}_{624}(0) | 0 \rangle$$

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$$\stackrel{t\to\infty}{\longrightarrow} \#\exp(-M_{Pb}t)$$

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- But...

An (exponentially hard)^2 problem?

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- Complexity: number of Wick contractions = (A+Z)!(2A-Z)!

$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

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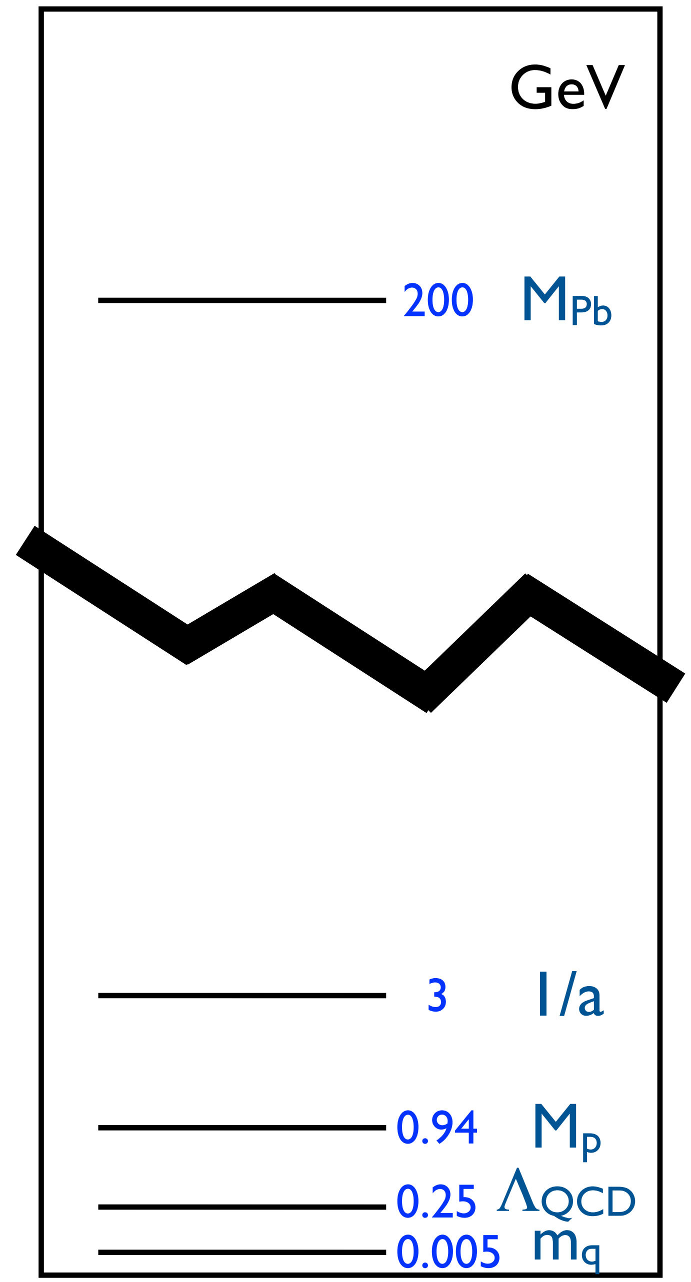

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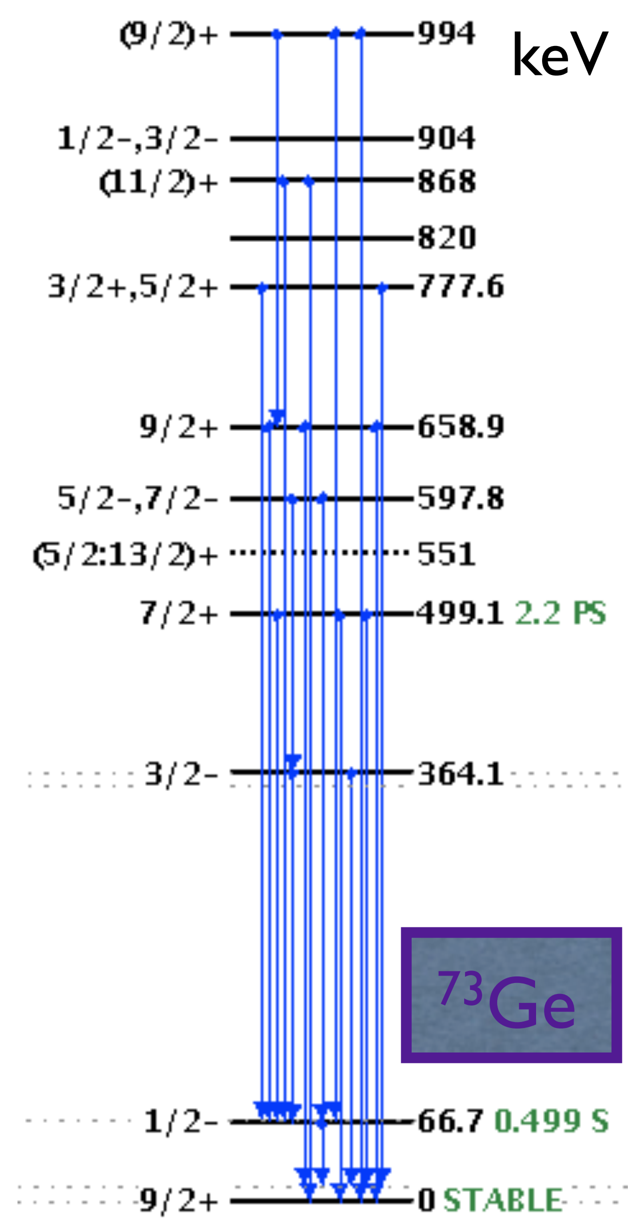

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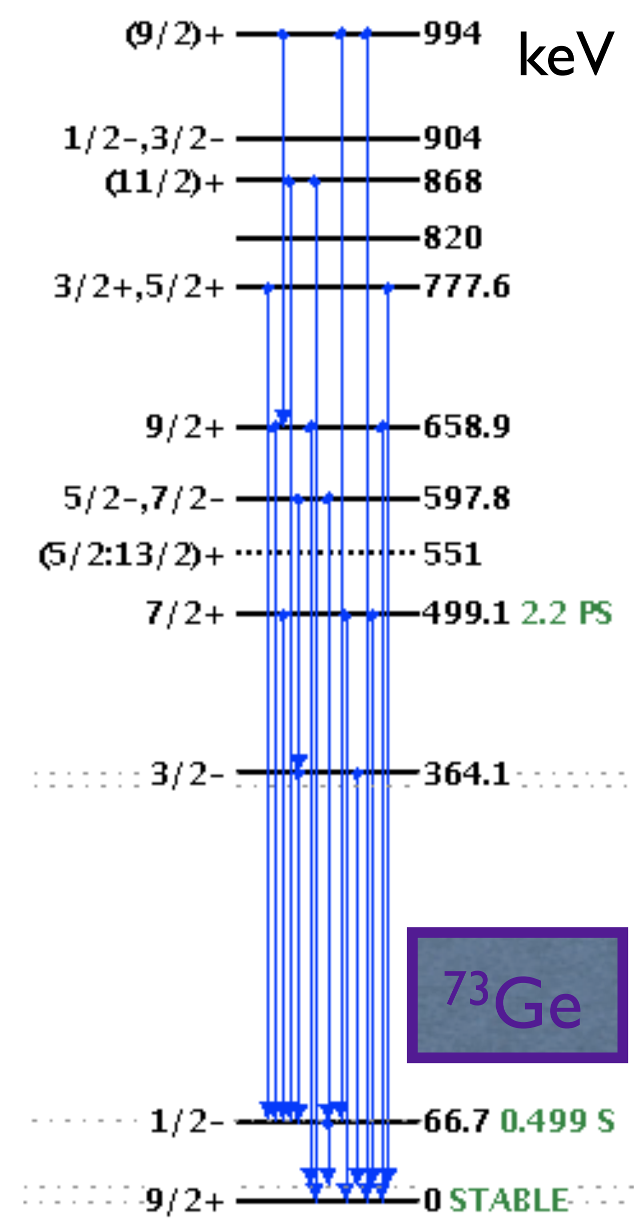
- Dynamical range of scales (numerical precision important)
- Small energy splittings and large objects
- Importance sampling: statistical noise exponentially increases with A



The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically
- Variance in single nucleon correlator (C) determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

- For nucleon:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi)t\right]$$

- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right]$$

[Lepage '89]

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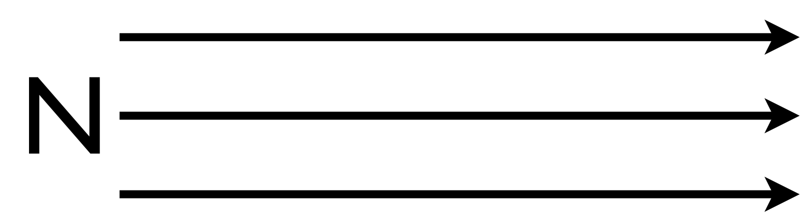

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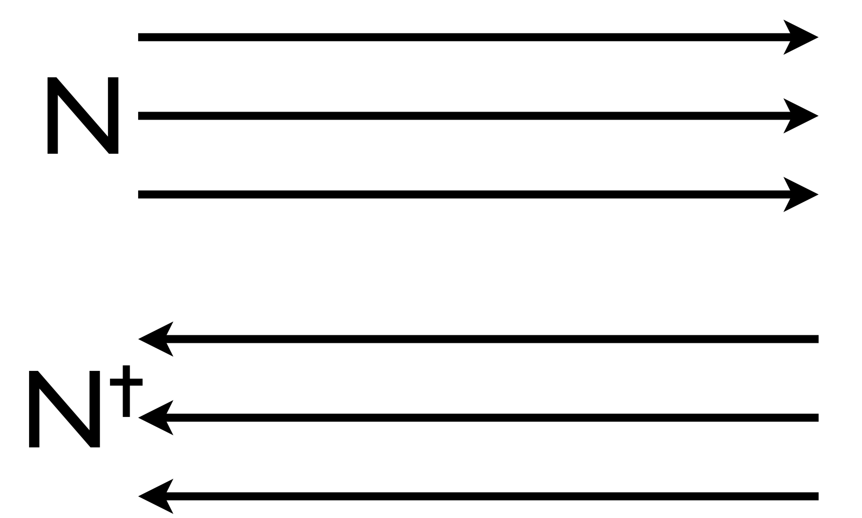

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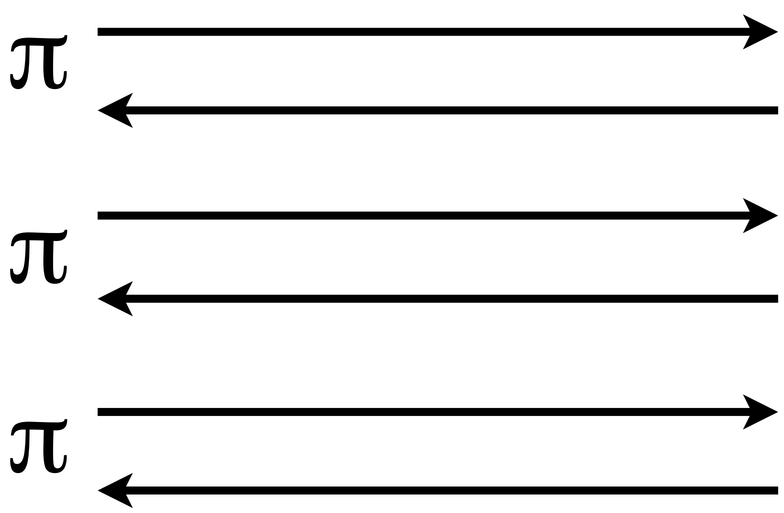

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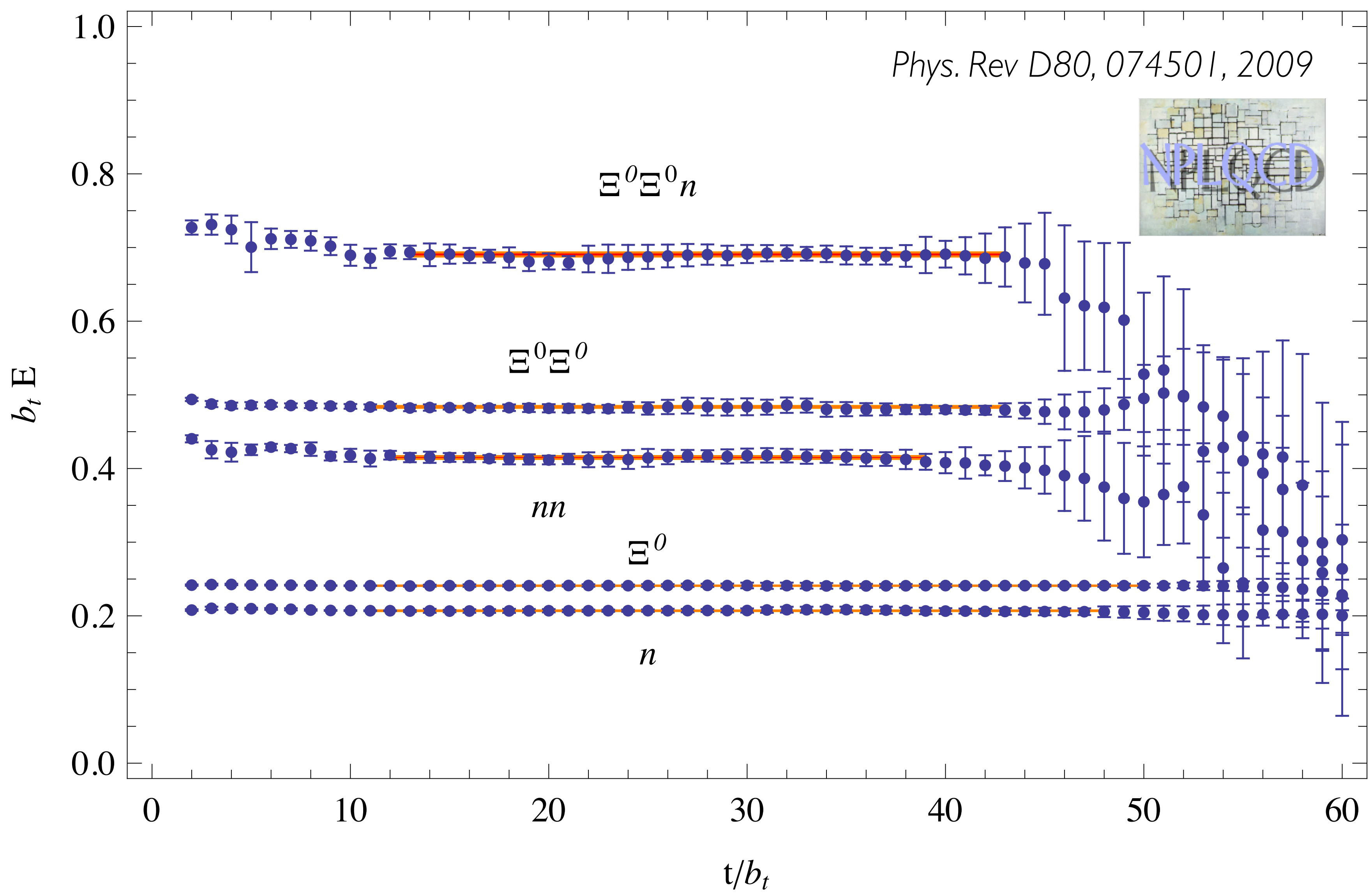
[Lepage '89]

The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

The trouble with baryons

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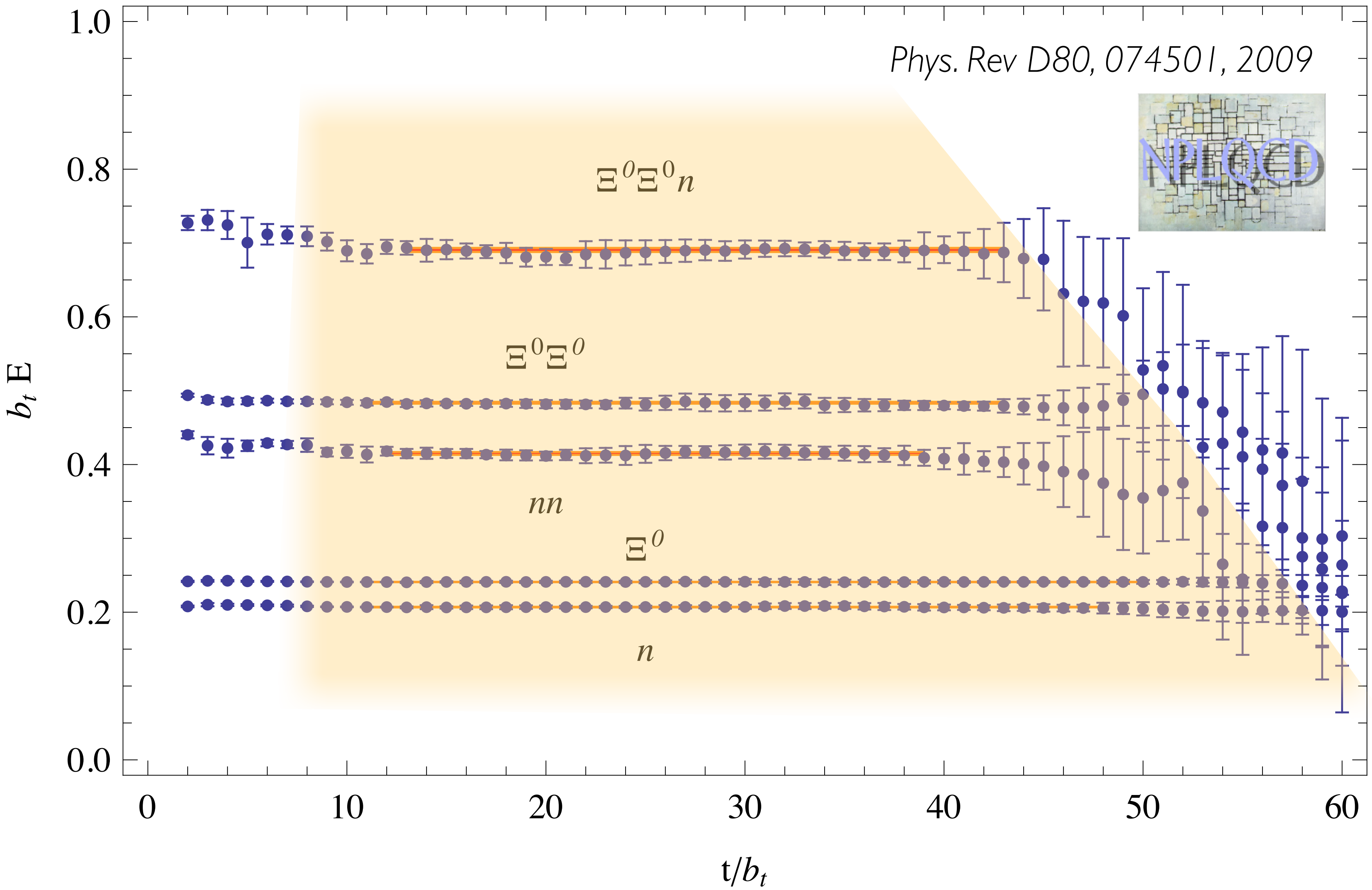

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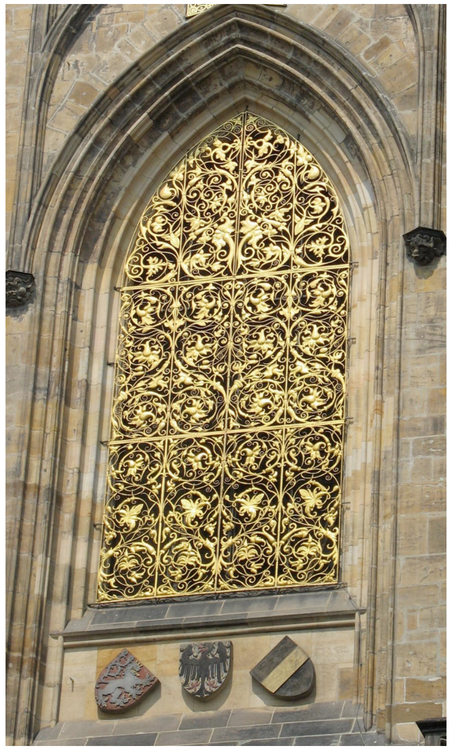

Golden window of time-slices where signal/noise const

No? trouble with baryons

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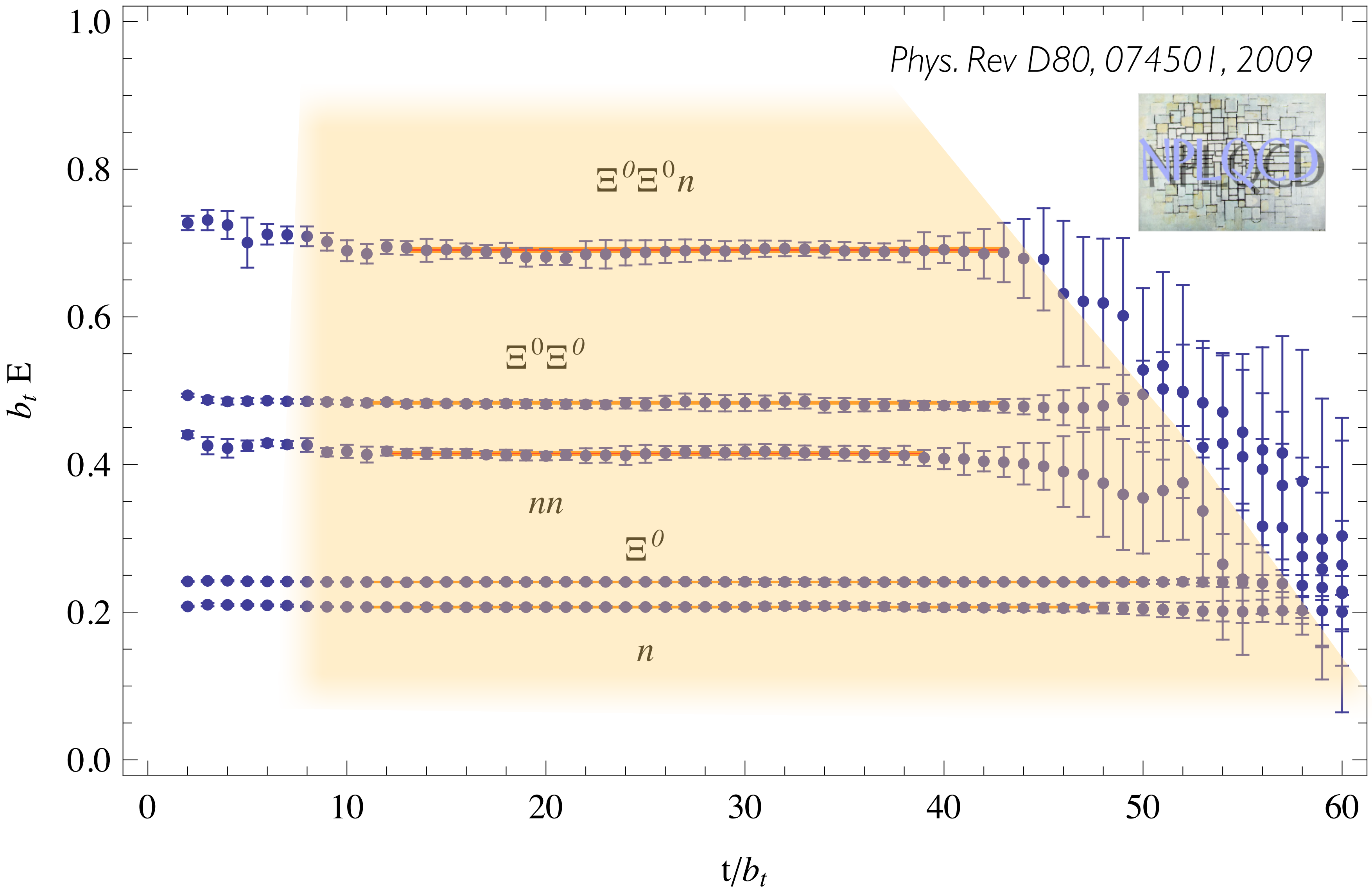

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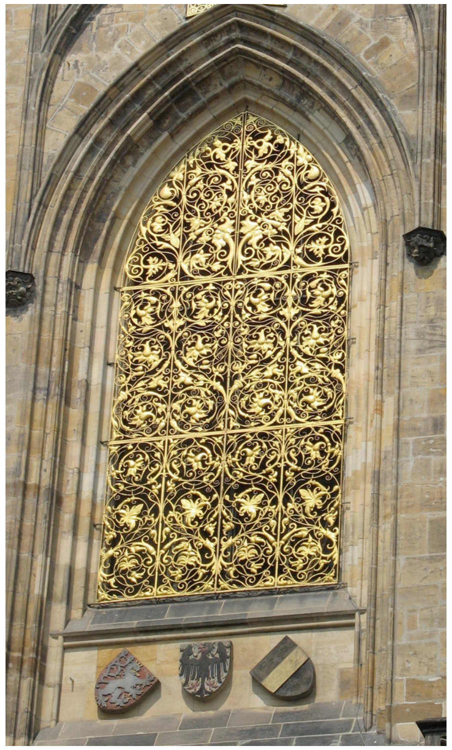

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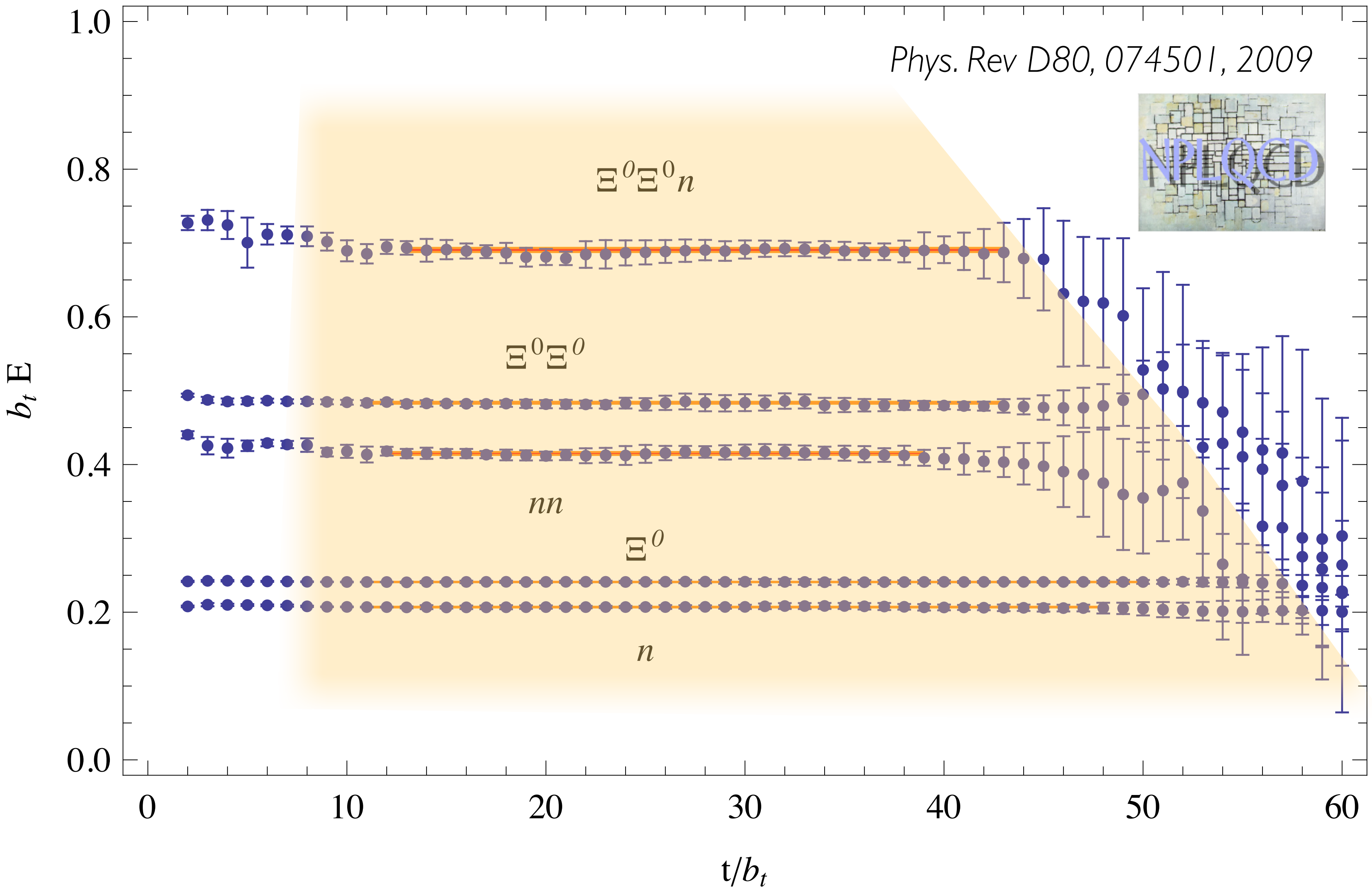

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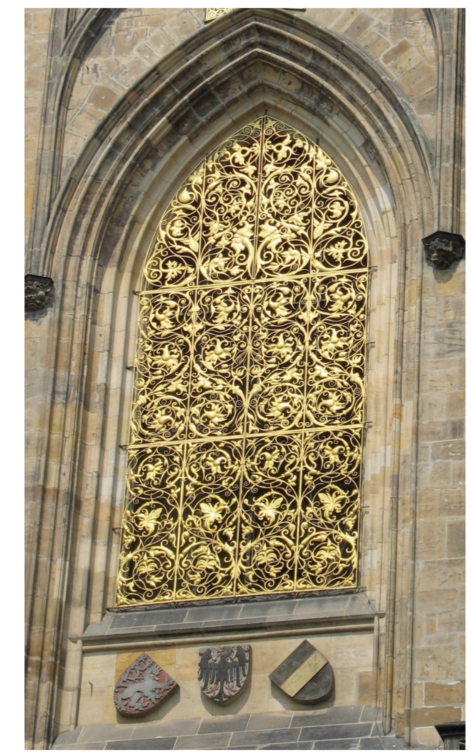
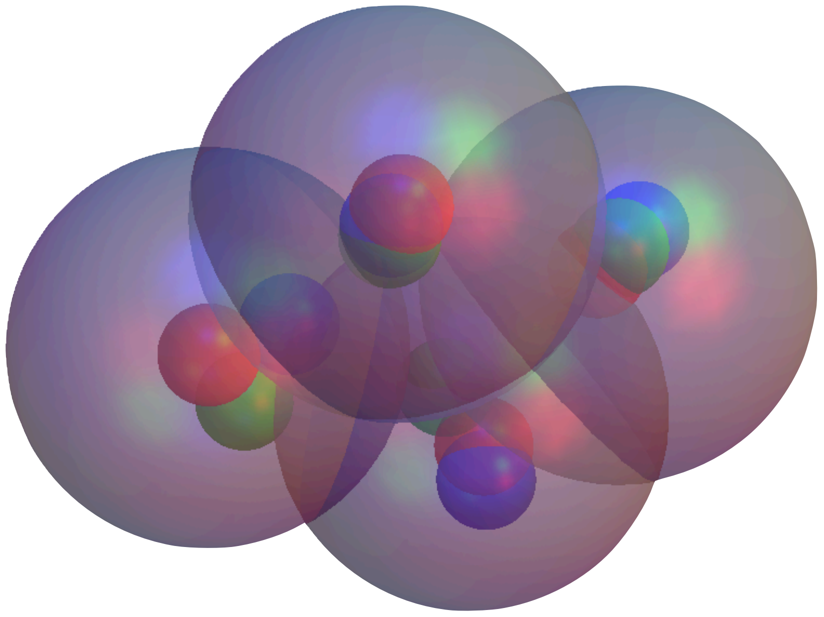


Golden window of time-slices where signal/noise const

Interpolator choice can be used to suppress noise

Multi-baryon systems

- Showed that the study of nuclei was feasible
  - Contractions still demanding
- Recent studies
  - NPLQCD
  - PACS-CS
  - HALQCD
- Resurgence in development of formalism to understand what is measured



Bound states at finite volume

- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M} \frac{1}{p \cot \delta(p) - ip}$$

scattering phase shift

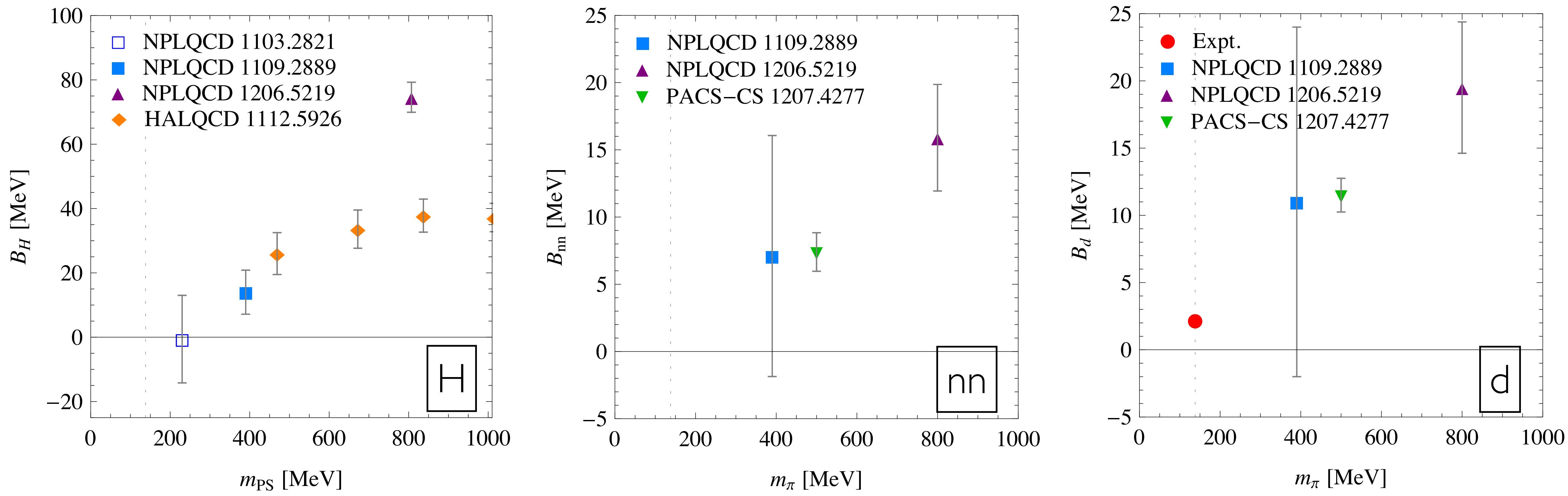
bound state at $p^2 = -\gamma^2$ when $\cot \delta(i\gamma) = i$

- Scattering amplitude in finite volume (Lüscher method)

$$\cot \delta(i\kappa) = i - i \sum_{\vec{m} \neq 0} \frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L \to \infty} \gamma$$

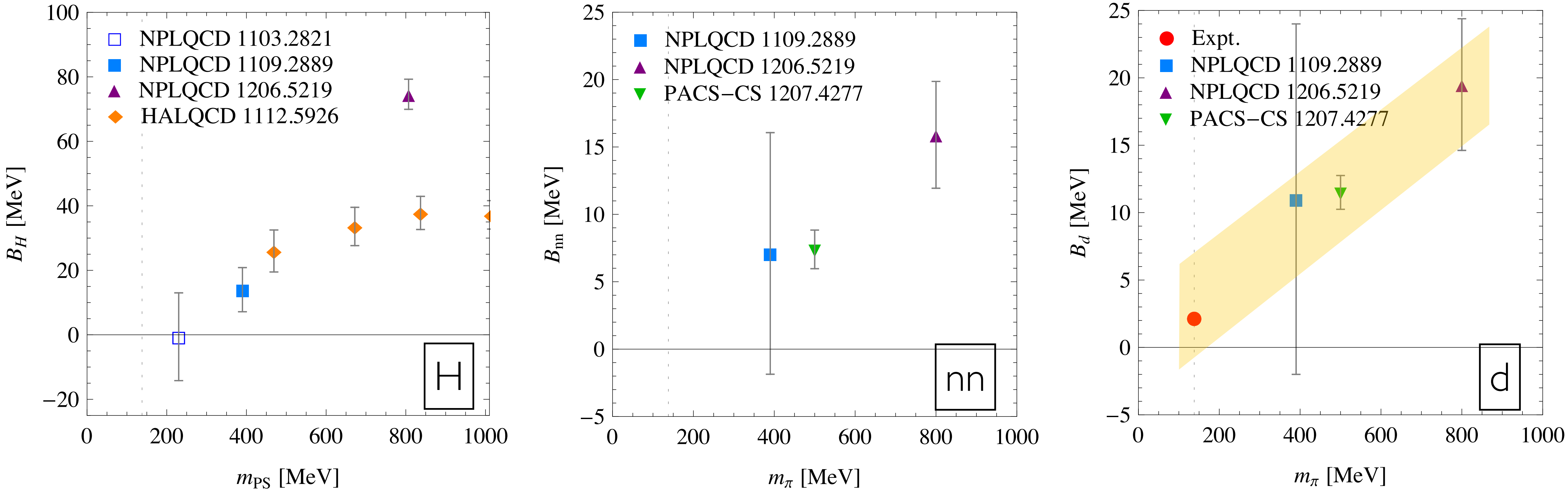
  - Need multiple volumes
  - Volume dependence set by binding momentum
  - More complicated for n>2 body bound states

Dibaryons



- H dibaryon (first B=2 bound state seen), di-neutron and deuteron
- More exotic channels also considered (**ΞΞ** and **ΩΩ**)
- More work needed at lighter masses

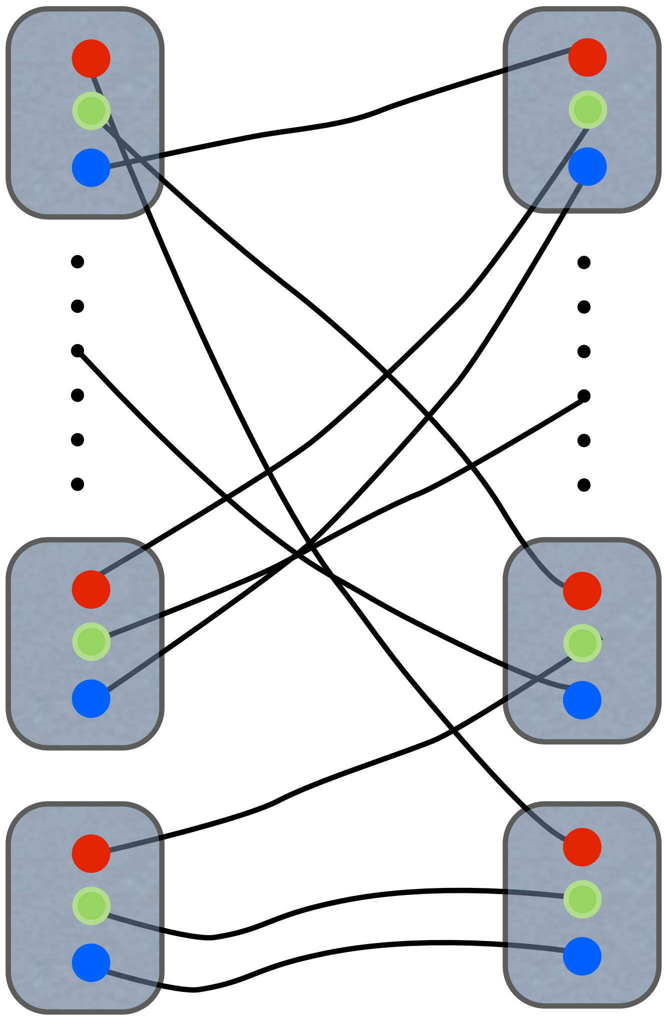
Dibaryons



- H dibaryon (first B=2 bound state seen), di-neutron and deuteron
- More exotic channels also considered ($\Xi\Xi$ and $\Omega\Omega$)
- More work needed at lighter masses

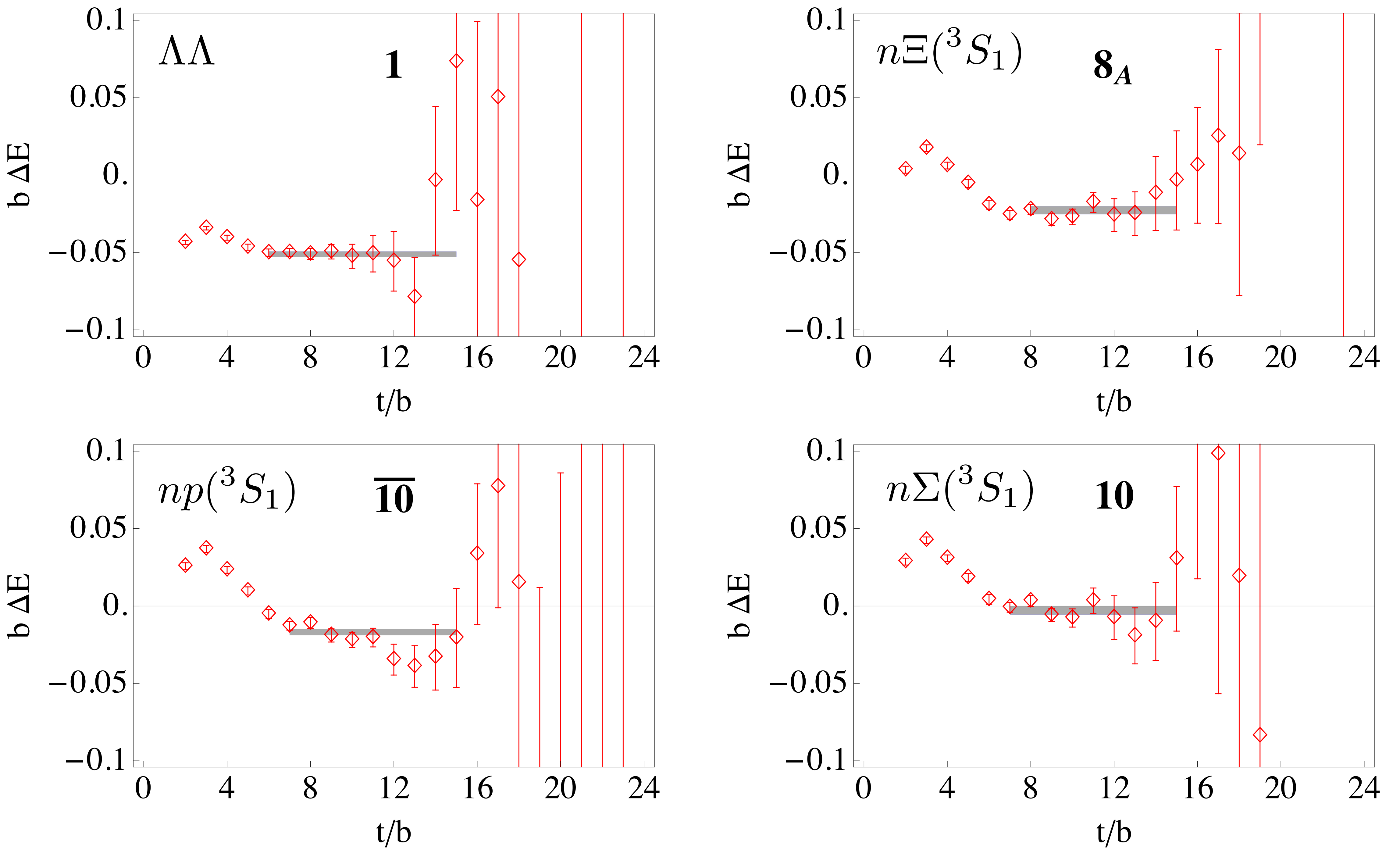
Multi-baryon systems

- Many baryon correlator construction is messy and expensive, but now feasible
    - Efficient contraction algorithms developed in many-pion studies (up to 72 pions!) [WD & M. Savage; WD,, K Orginos, Z. Shi; T. Doi & M. Endres.; WD, K Orginos]
- Enables study of few (and many) baryon systems
- NPLQCD collaboration
    - Unphysical SU(3) symmetric world @ m_s^{phys}
    - Multiple big volumes, single lattice spacing



NB: SU(3) symmetry leads to unphysical degeneracies

Nuclei (A=2)



NPLQCD Phys.Rev. D87 (2013) 034506

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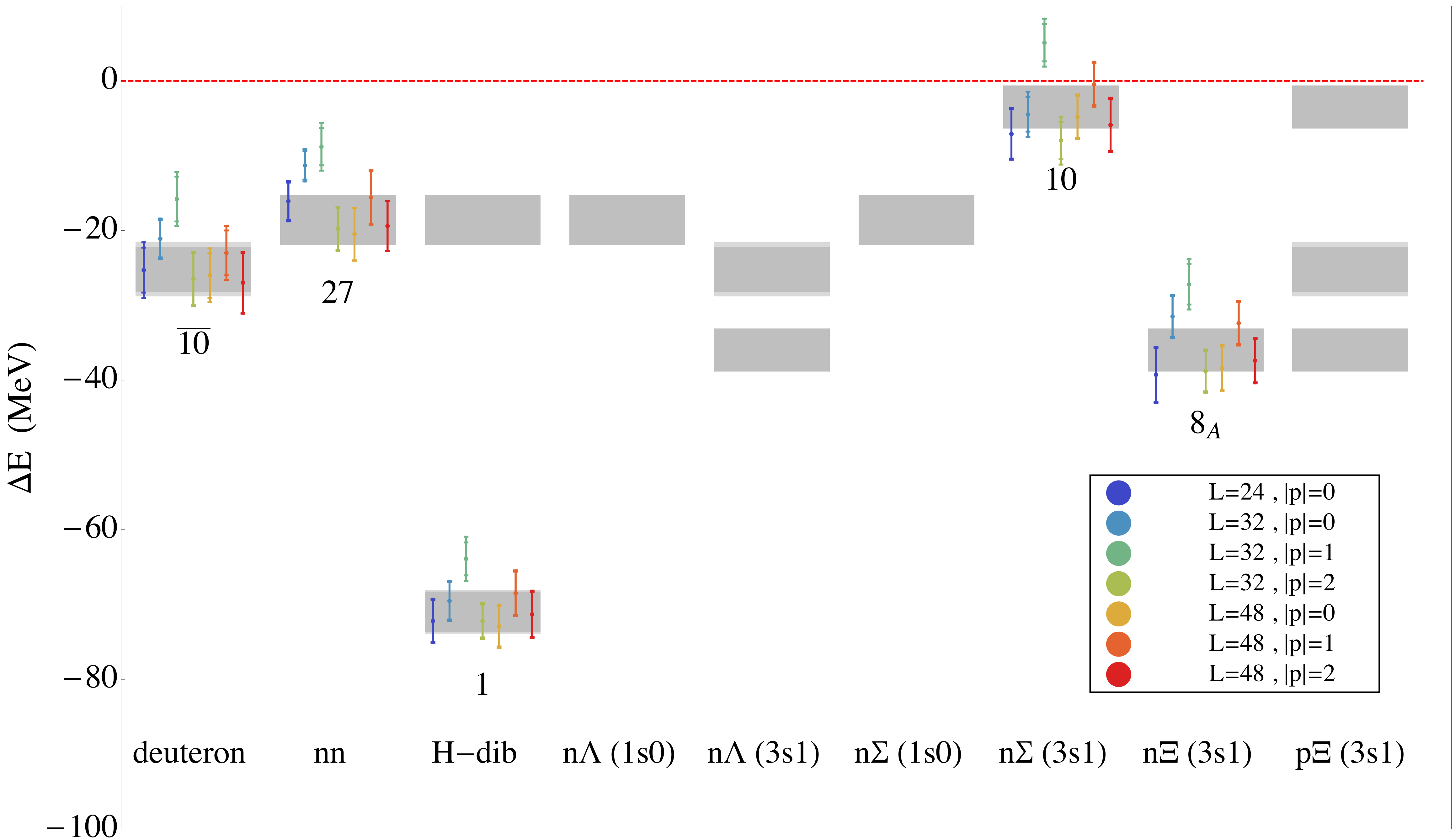

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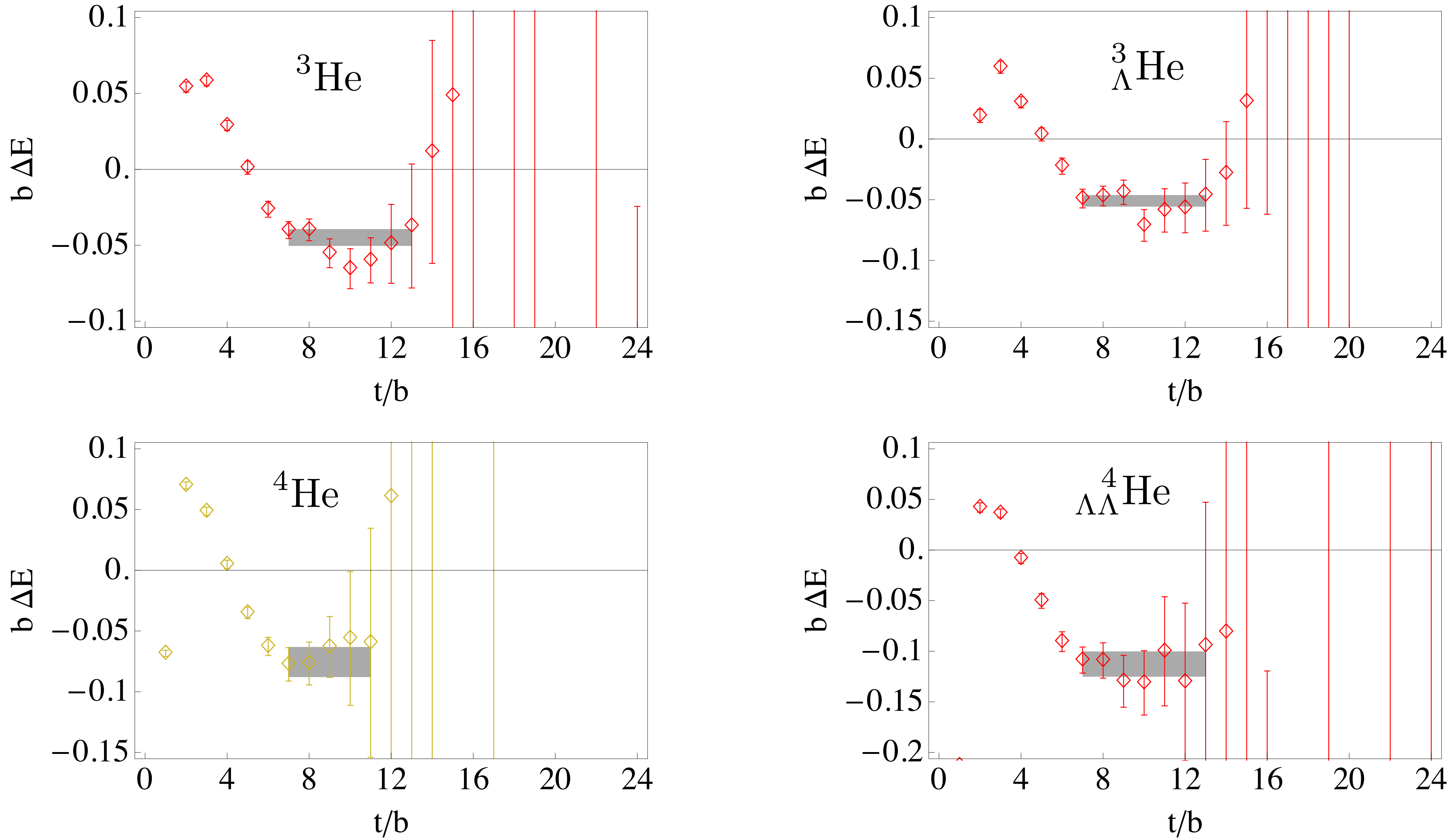
Nuclei (A=3,4)



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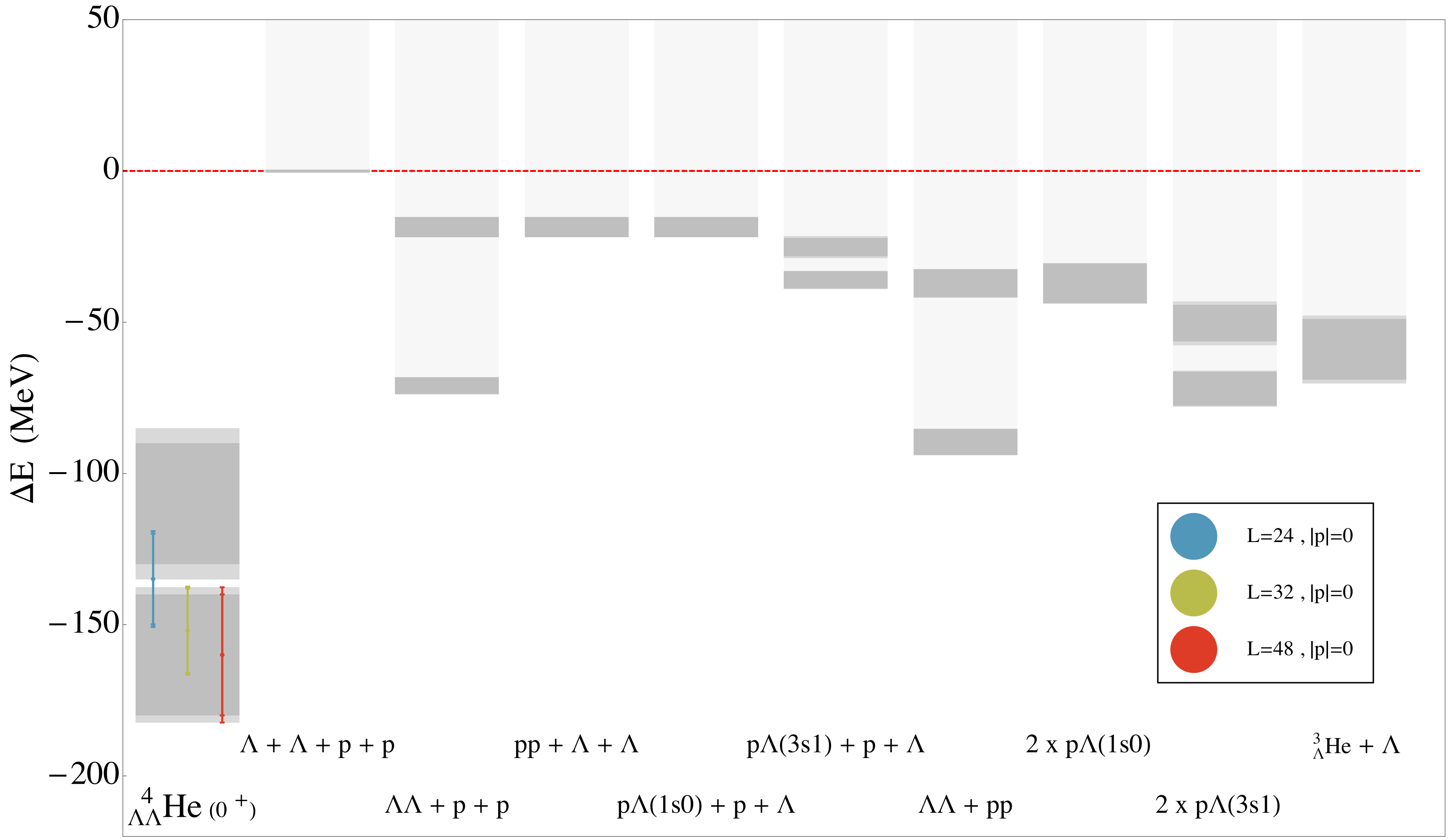
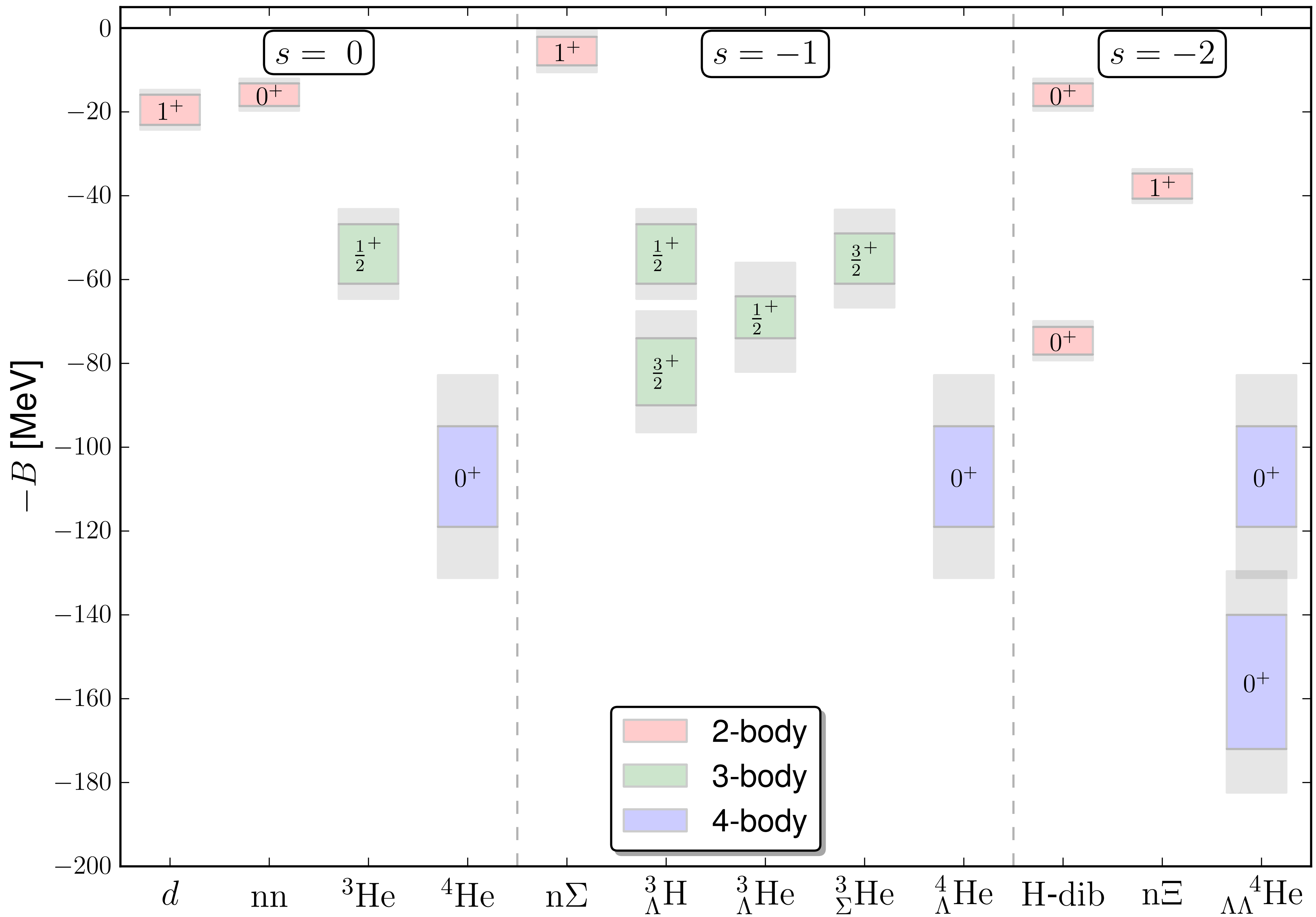
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state



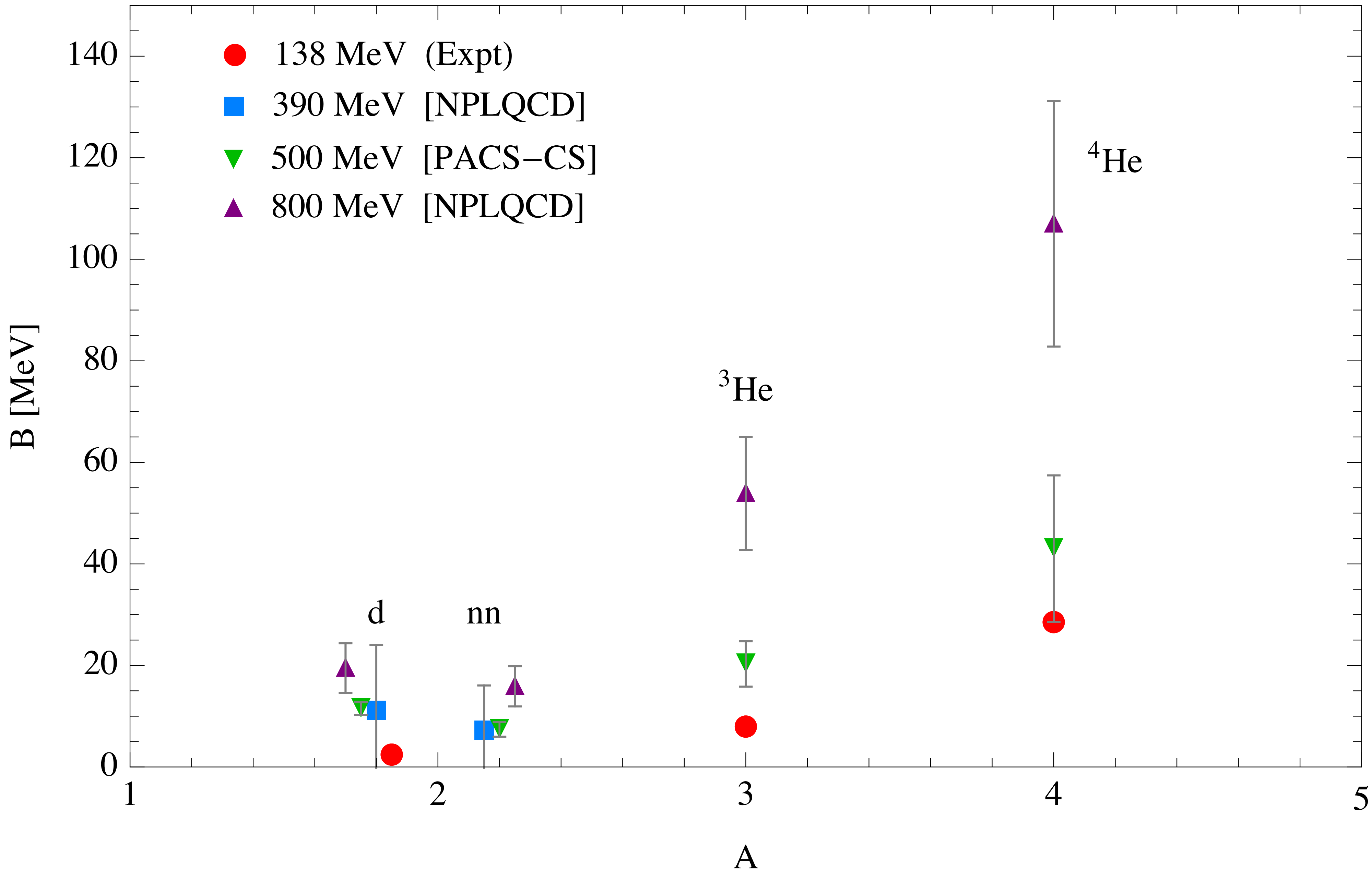
NPLQCD Phys.Rev. D87 (2013) 034506

Nuclei (A=2,3,4)



NPLQCD Phys.Rev. D87 (2013) 034506

Periodic table



- Significant competition from PACS-CS and HALQCD (Japan) Also from groups in Germany

Nuclei (A=4,8,12,...)

Quark determinant based contraction method

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

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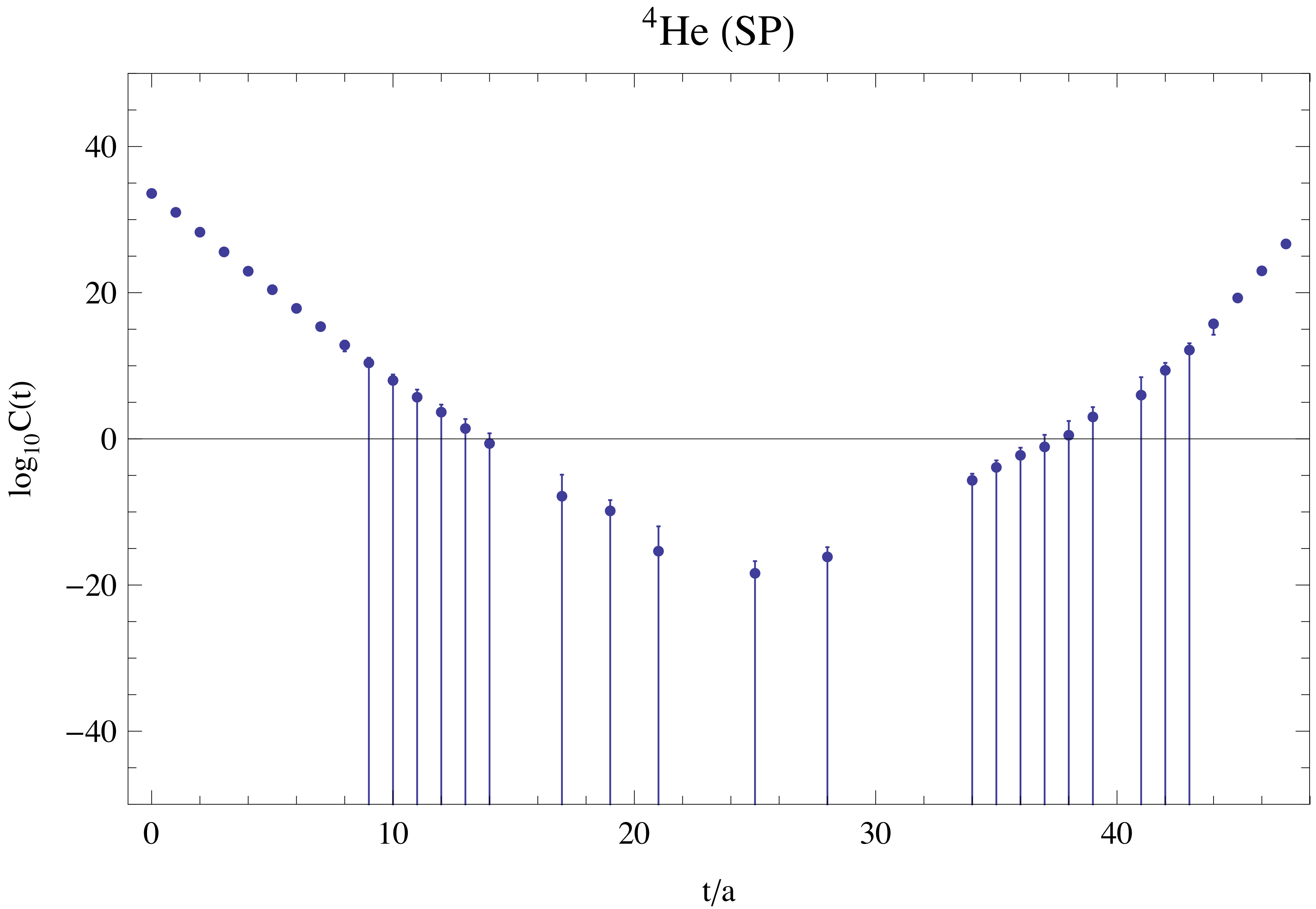

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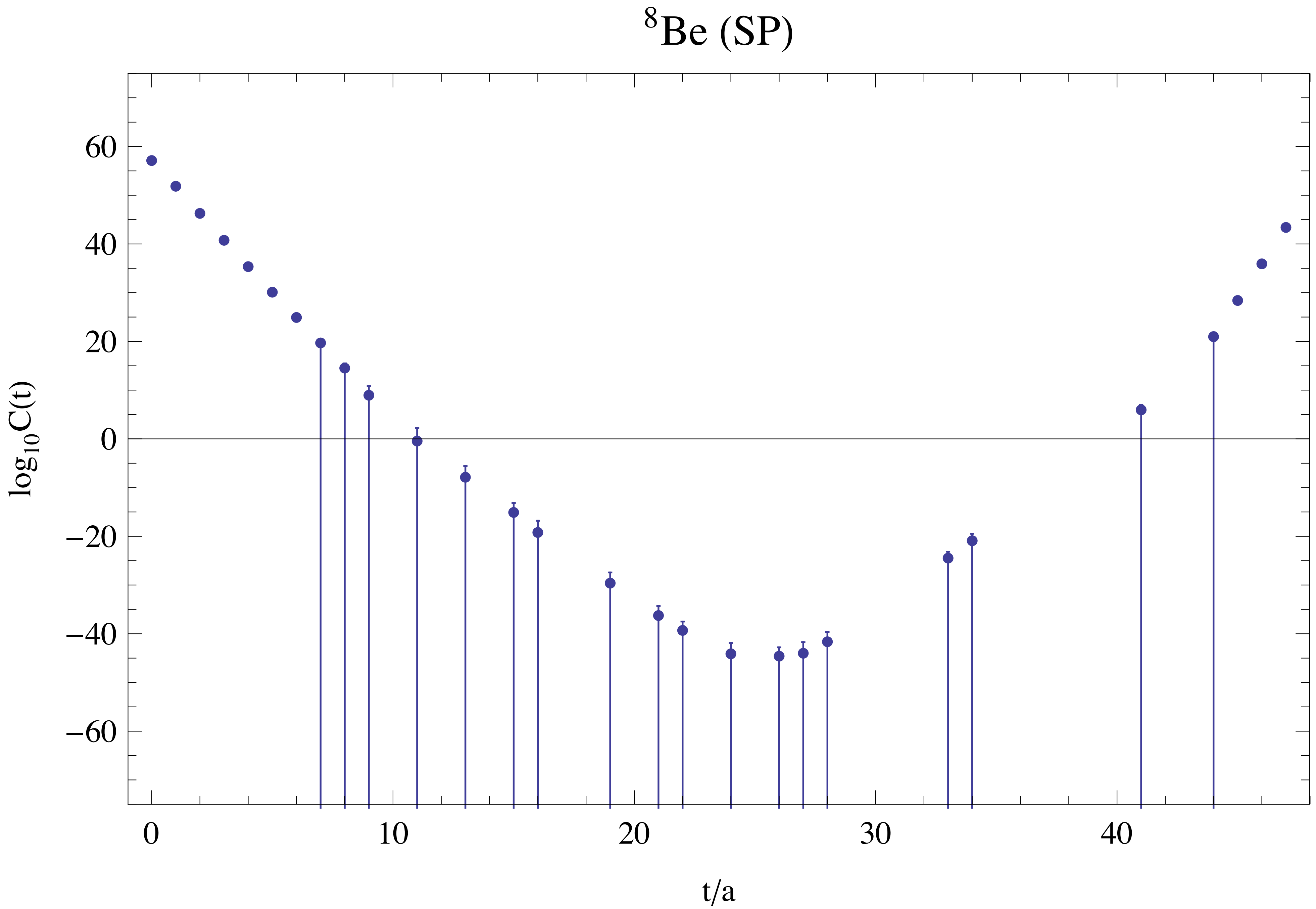

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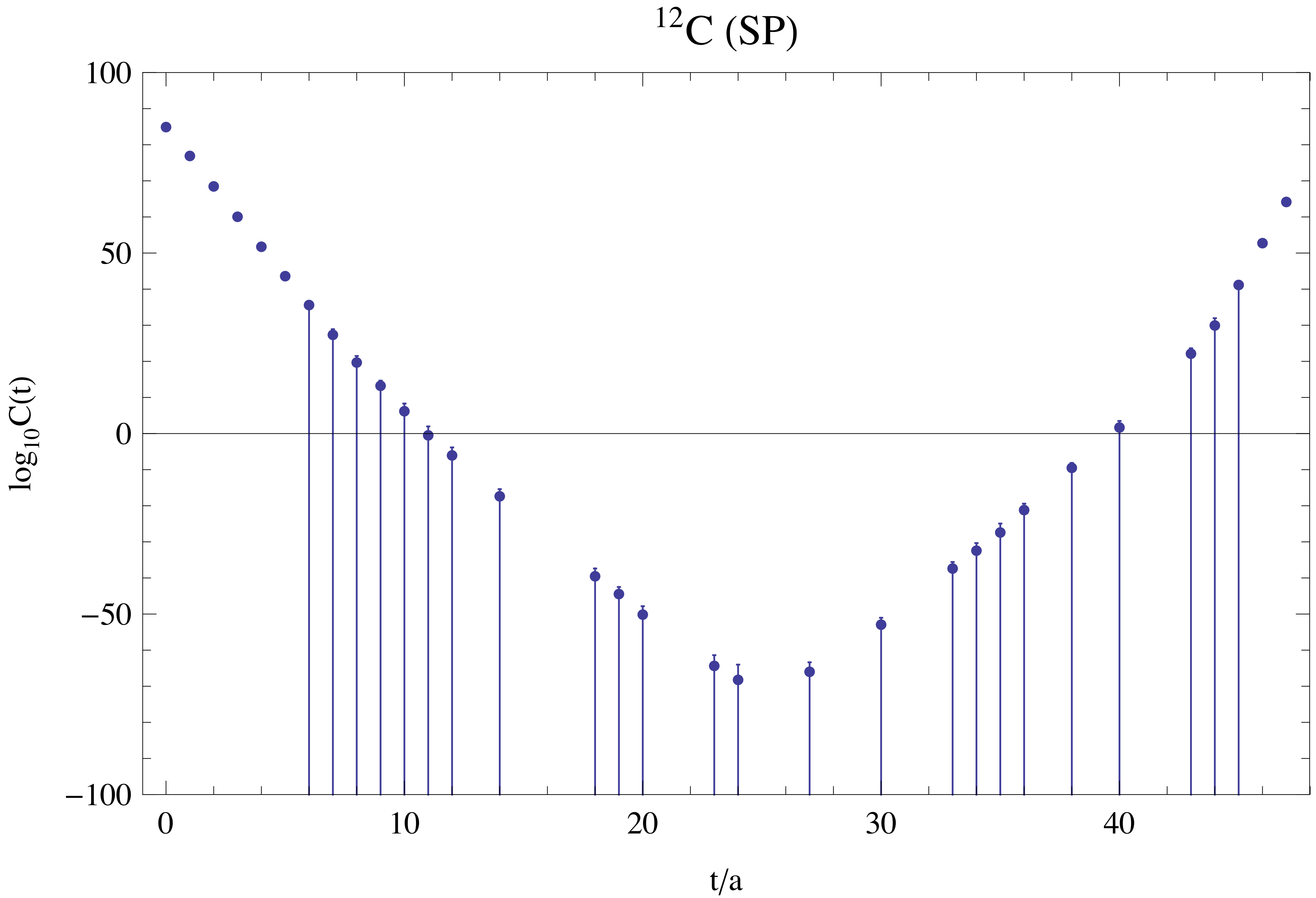

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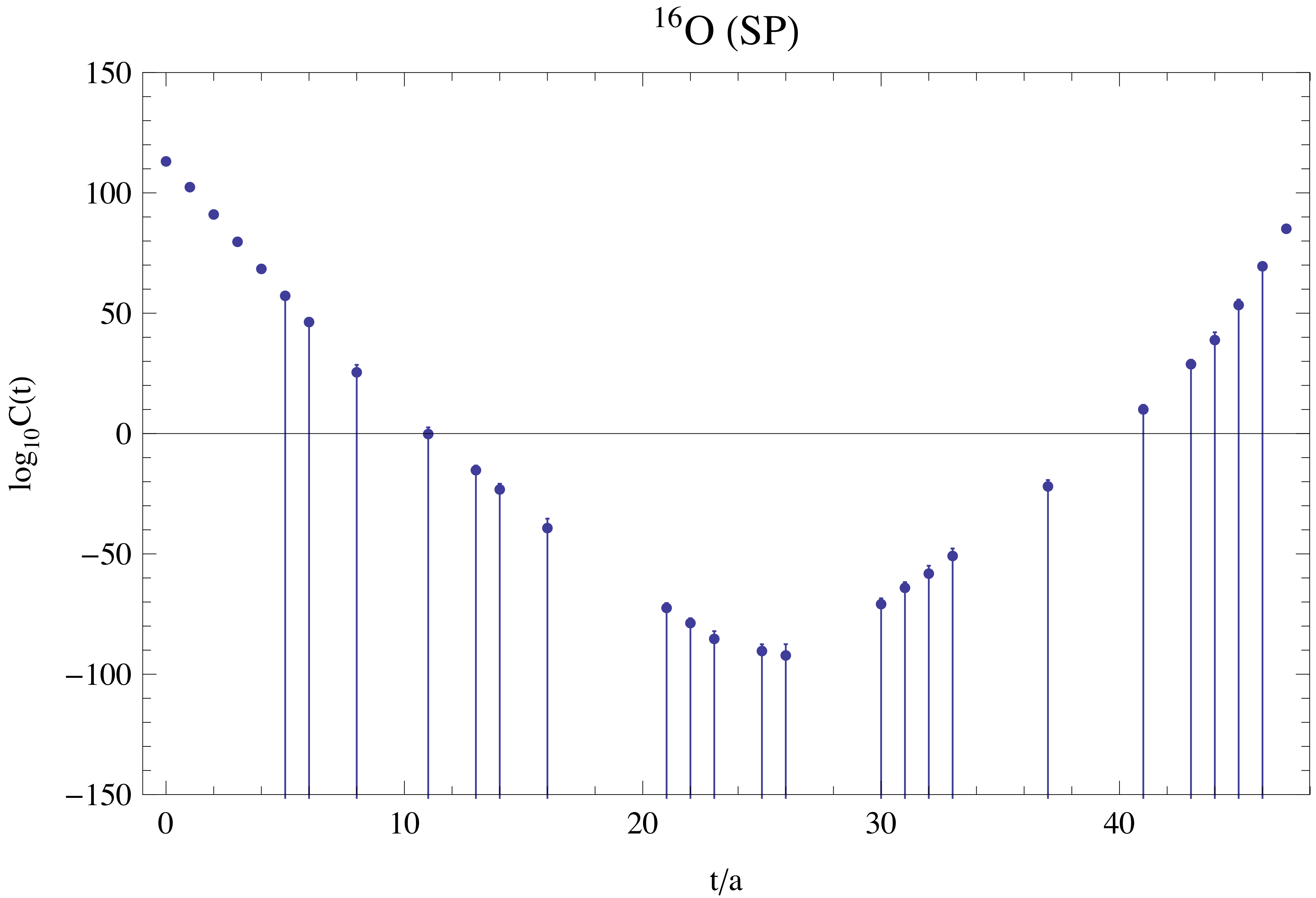

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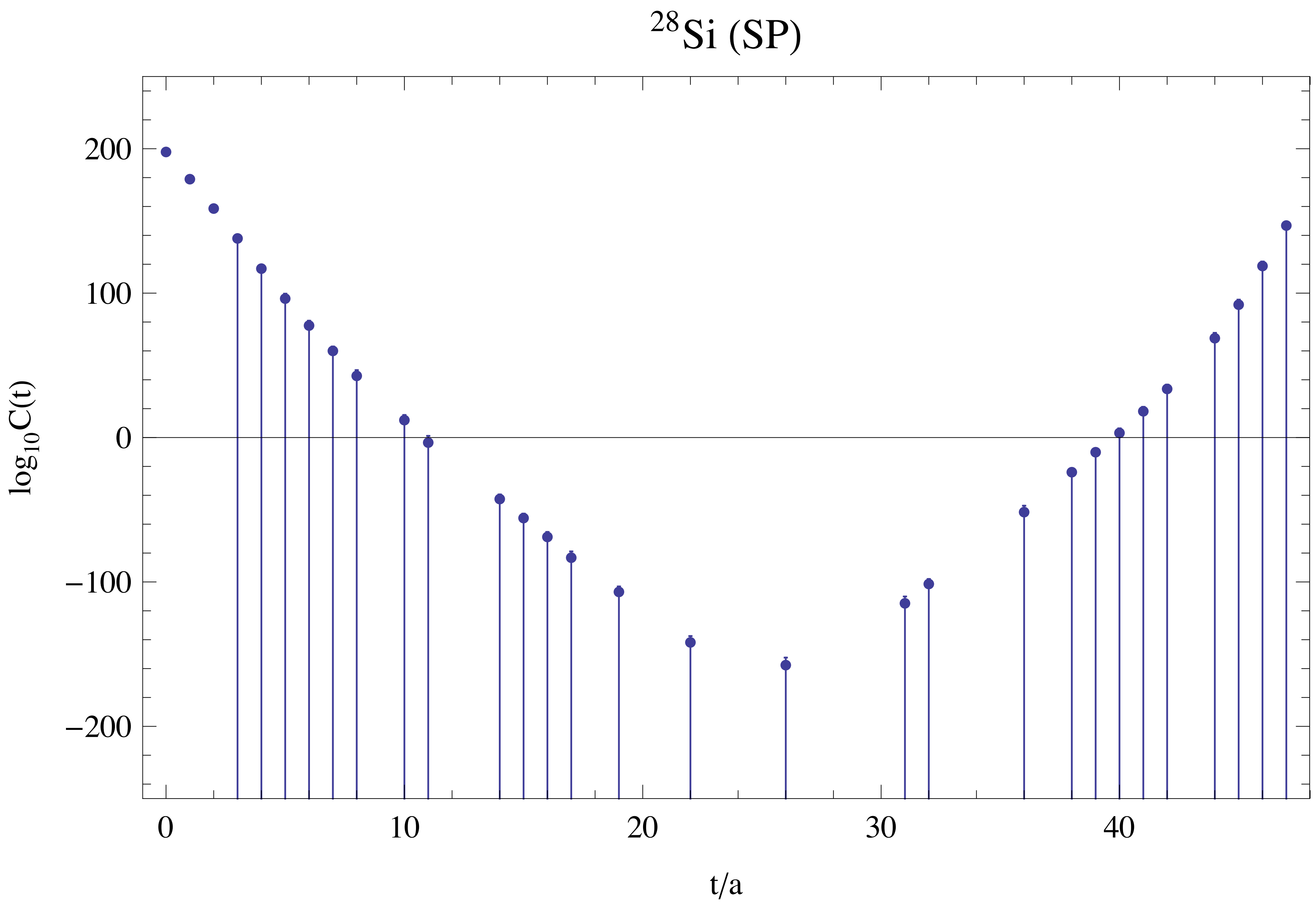

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Hyperon-nucleon interactions

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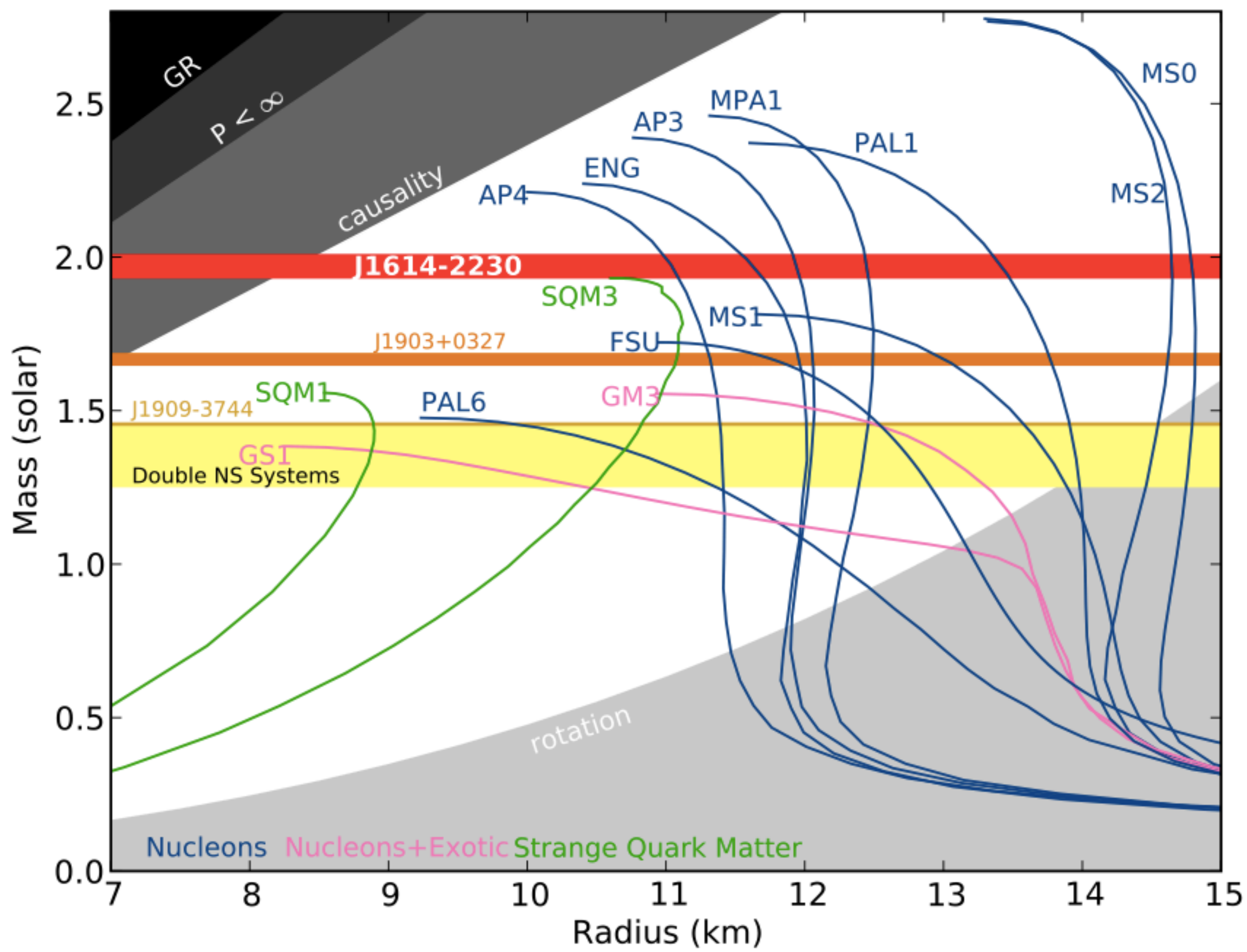
- Observation of 1.97 $M_{\odot}$ n-star [Demorest et al., Nature, 2010]
  "effectively rules out the presence of hyperons, bosons, or free quarks"



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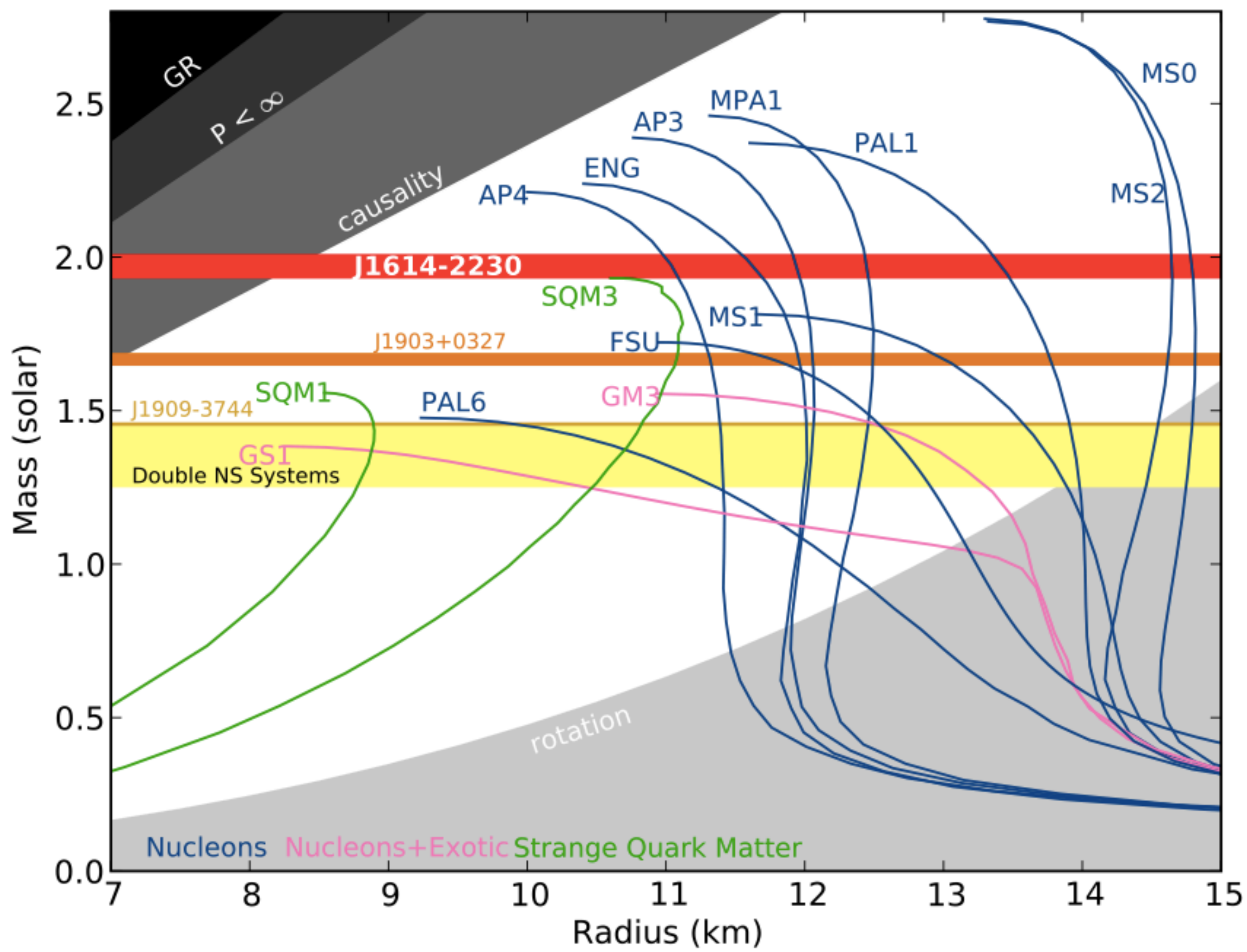
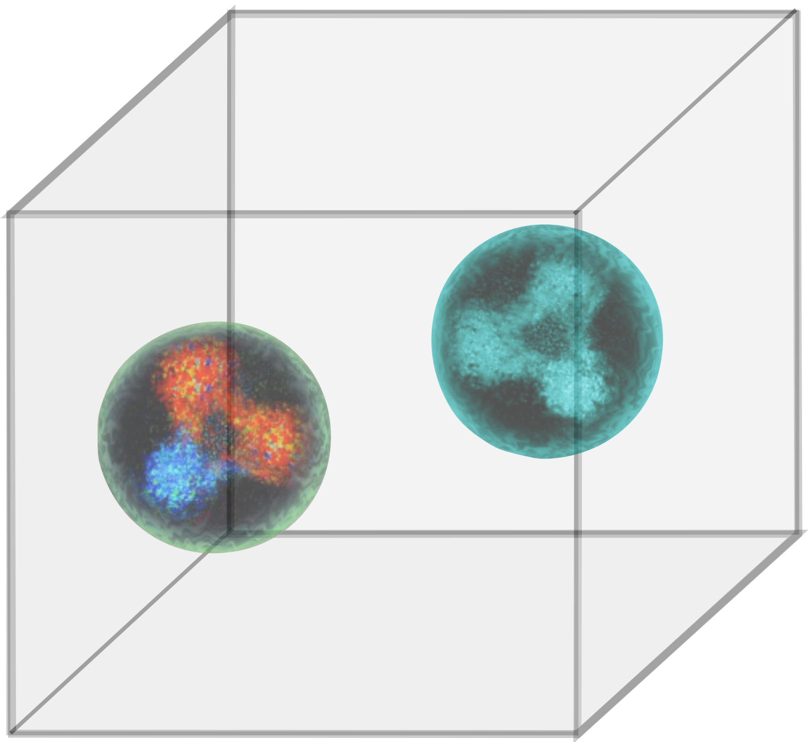


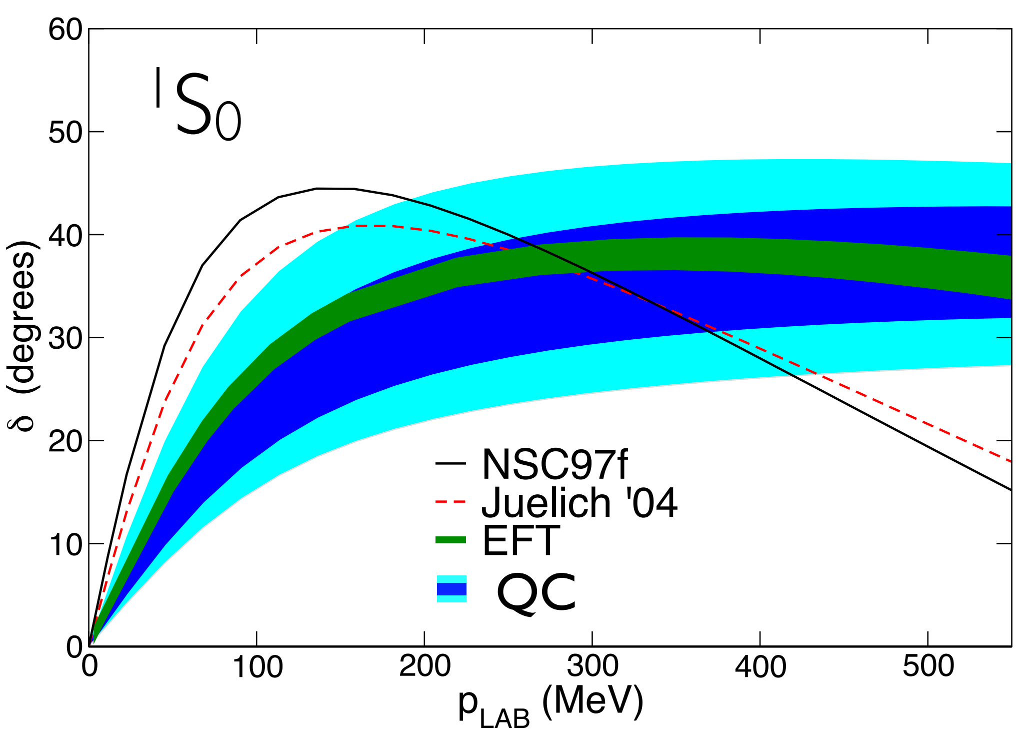
- Relies significantly on poorly known hadronic interactions at high density: hyperon-nucleon, nnn, ...

Σ^{-} *n interactions*



- Scattering in Euclidean space?
- Two particles in a box
  - Eigen-energies depend on interactions (Lüscher method)

$$E_{BB},\ E_B \longrightarrow q_{\rm scat} \longrightarrow p\ \cot\delta(p)|_{p=q_{\rm scat}}$$



- Use finite volume energy levels to determine hyperon-nucleon phase shift from QCD
- Match to effective field theory to extract phase shift at physical mass

Phys. Rev. Lett. 109 (2012) 172001

Σ^- n interactions

- Influence on EoS is complicated
  - Crude approx: Fumi's theorem

$$\Delta E = -\frac{1}{\pi\mu}\int_0^{k_f} dk\, k \left[\frac{3}{2}\delta_{^3S_1}(k) + \frac{1}{2}\delta_{^1S_0}(k) \right]$$

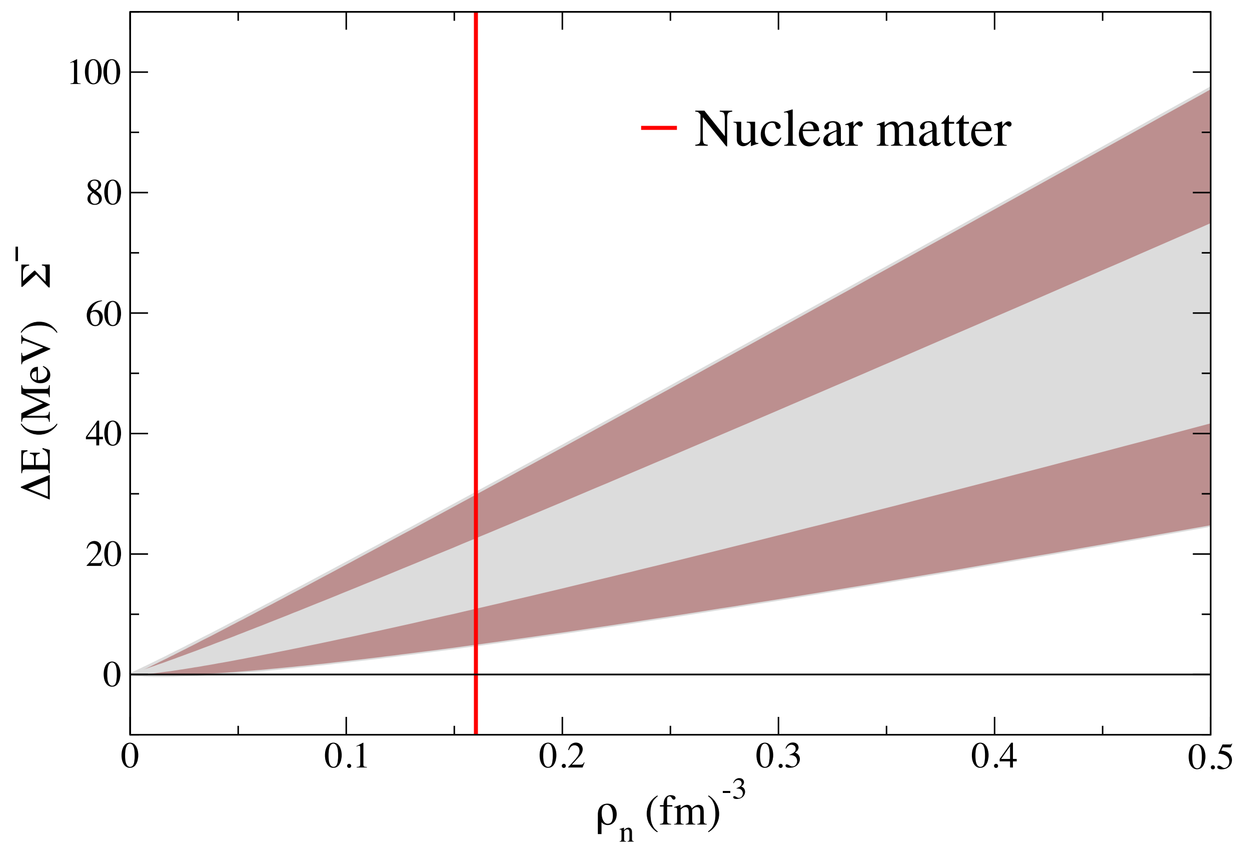
- For $\rho_n \sim 0.4 \text{ fm}^{-3}$,

$$\mu_n + \mu_{e^-} \sim 1290 \text{ MeV}$$

- If

$$\mu_{\Sigma^-} = M_\Sigma + \Delta E \lesssim 1290 \text{ MeV}$$

then Σ^-s may be relevant to n-star structure



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NN Phase shifts

- Measurement of multiple energy levels allows extraction of phase shifts

- Ex: $^3S_1$ phase shift at m_π=800 MeV

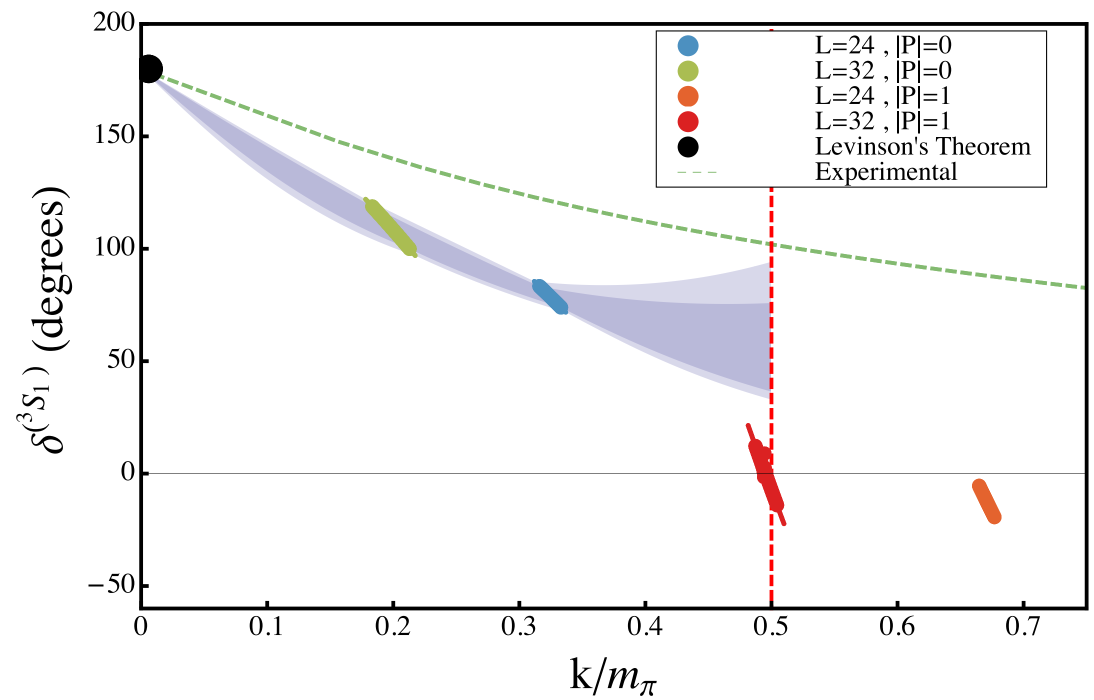
$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12} \text{ fm} \qquad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084} \text{ fm}$$

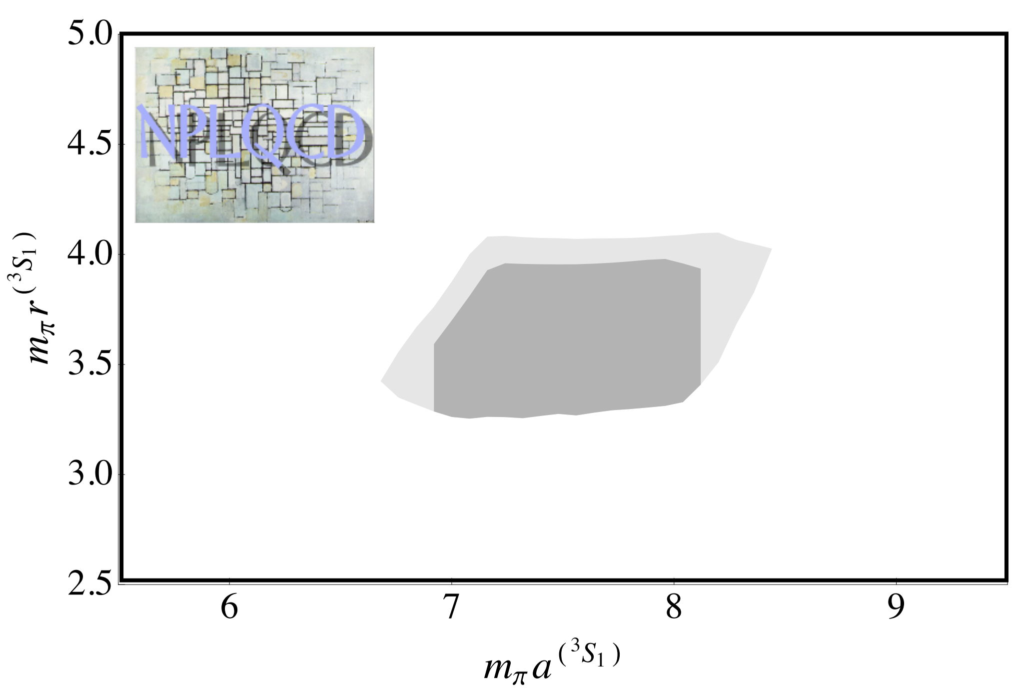
$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

- *c.f.* fine-tuning of NN at physical mass

- Wigner SU(4) symmetry





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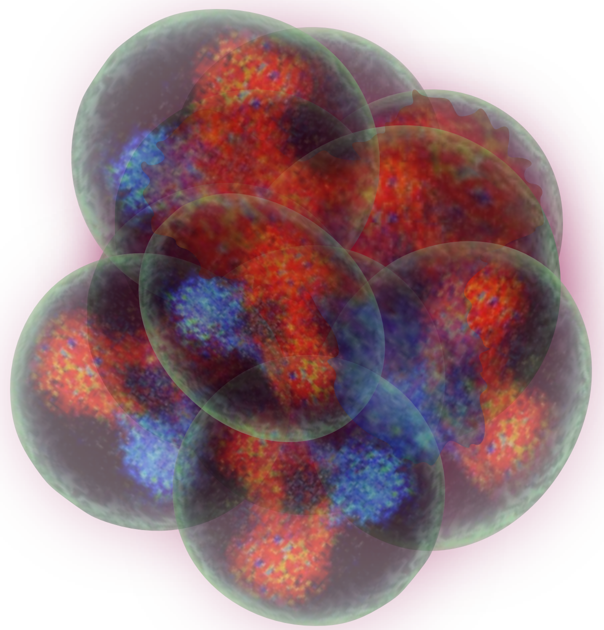
Future Prospects

- What does the future hold?
  - Physical quark masses, isospin breaking, E&M(?)
  - Precision YN, YY phase shifts
  - *p*-shell and larger nuclei
  - Three body information: nnn, YNN, ...
  - Properties of light nuclei (moments/structure) and electroweak interactions
  - Nuclear reactions(?): eg d+d in ^{4}He channel

From quarks to nuclei

- Nuclear physics is an emergent phenomenon of the Standard Model
- *What does it take to make this a quantitative statement?*
    - Big computers and good ideas
- New endeavor in LQCD with a promising future
    - Strong connections to experimental programs at JLab, FRIB, and FAIR
    - Answer questions that experiments have not and can not



[FIN]

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