Weak Matrix Elements: Goals and Computational Challenges

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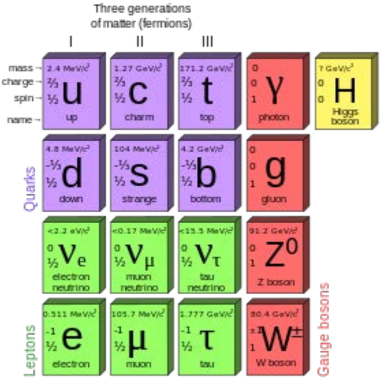
Outline

1. Goals of contemporary weak matrix element calculations
2. Computational challenges
3. Conclusions

Standard Model

- Well established, but incomplete.
- We know there is more to discover.
    - Dark matter
    - Matter/antimatter asymmetry

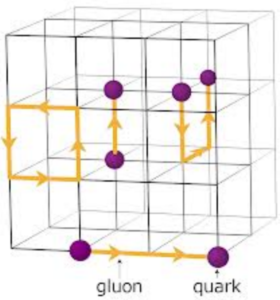


Intensity Frontier

- Major goal of the US High Energy Physics program:
  Search for physics beyond the Standard Model. AKA "New Physics."
- High precision experimental effort to detect discrepancies with SM predictions.
- Lattice calculations are needed to support this effort
- I will give some examples and then discuss computational challenges.

Why lattice QCD?

- Fully nonperturbative.
- Fully QCD (all relevant sea quarks)
- *ab initio i.e.* can be indefinitely improved by taking the lattice spacing to zero and the box size to infinity.



[Figure credit Guido Cossu]

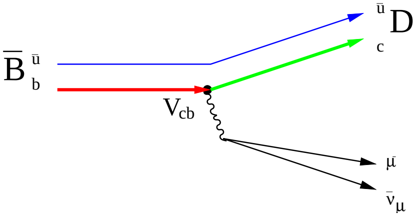
CKM Matrix: Unitarity

$$\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\
 & K \to \pi\ell\nu & \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle &
\end{pmatrix}$$

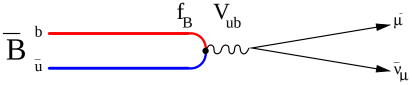
- This Standard Model 3×3 unitary matrix (bold elements) controls transitions between (decays of) quark "flavors".
- Also shown are processes used in experiment and theory to measure their magnitudes.
- 2008 Physics Nobel Prize: Nambu, Kobayashi, Maskawa

Examples of weak processes studied

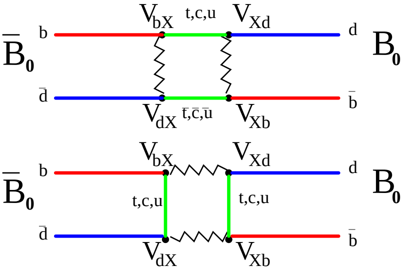
Semileptonic



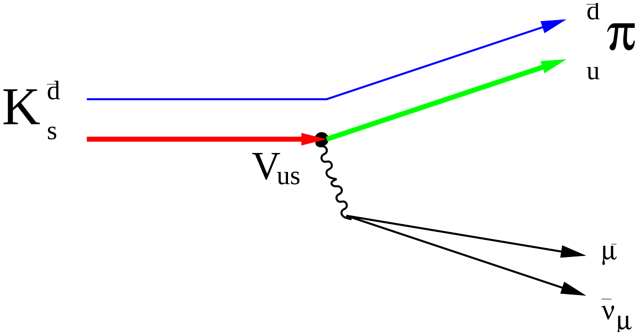
Leptonic (decay constant f_B)



$B - \bar{B}$ Mixing



High precision result: $K \to \pi\ell\nu$



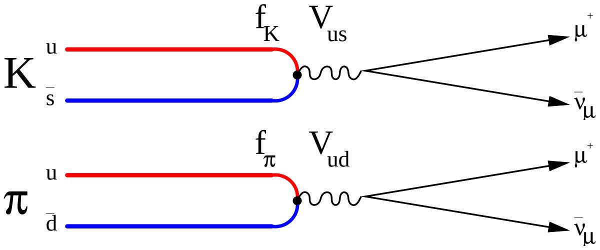
- New FNAL/MILC result [arXiV:1212.4993]

$$|V_{us}| = 0.2238 \pm 0.0009_{\mathrm{thy}} \pm 0.0005_{\mathrm{expt}}$$

- Unitarity check

$$\Delta_{\mathrm{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008(6)\,.$$

High precision example: f_K/f_π

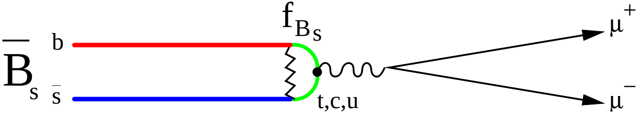


- New HPQCD result [arXiv:1303.1670]

$$f_{K^+}/f_{\pi^+} = 1.1916(21)$$

- To do better, we must now take into account electromagnetic effects

Beyond the Standard Model $B_s \to \mu^+\mu^-$



- The rare decay $B_s \to \mu^+\mu^-$ is considered one of the promising places to look for new physics.
- We need the decay constant f_{B_s} to get the Standard Model prediction.
- New HPQCD result [arXiv:1302.2644]

$$f_{B_s} = 224(5)\mathrm{MeV}$$

- The Standard Model predicts (Buras *et al* [arXiv:1208.0934])

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (3.17 \pm 0.15_{\mathrm{other}} \pm 0.09_{f_{B_s}}) \times 10^{-9}$$

- Error from f_{B_s} is now smaller than from other sources.
- First measurement (LHCb) [PRL **110**, 021801 (2012)]:

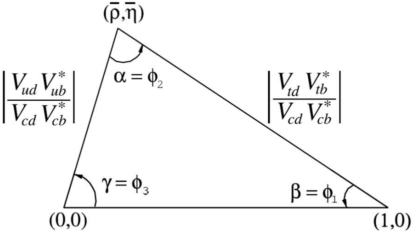
$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9}.$$

- So no evidence for new physics, yet.
- Smaller errors will come from LHCb.

CKM Matrix: Unitarity

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + 1 + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0$$



[Figure credit - PDG review]

Unitarity triangle

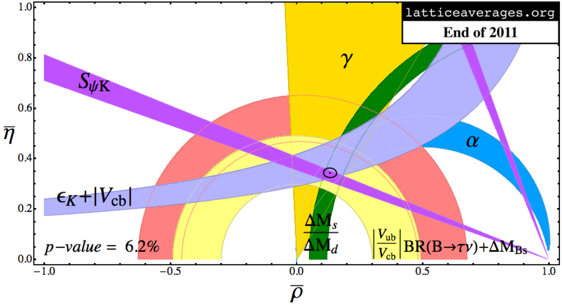


Figure: arXiv:1204.0791

- Each band corresponds to a combination of experiment and theory for some quantity.
- Width combines uncertainties in theory and experiment.
- Standard Model says bands must overlap at a single point. Here they disagree by about 3σ.
- But not enough to say we have something new here.
- Goal is to narrow the width of the error bands.
- Uncertainty of theory typically lags that of experiment.

$|V_{cb}|$ from $B \to D^* \ell \nu$

- As usual we get the CKM matrix element from the ratio of experiment to theory

$$|V_{cb}| = \frac{|V_{cb}|\mathcal{F}(1)_{\rm expt}}{\mathcal{F}(1)_{\rm thy}}$$

- The Heavy Flavor Averaging Group [arXiv:1207.1158] compiles results from an average of several recent experiments:

$$|V_{cb}|\mathcal{F}(1)_{\rm expt} = (35.90 \pm 0.45) \times 10^{-3}$$

- The FNAL/MILC Collaborations reported preliminary results [arXiv:1011.2166] (final result very soon):

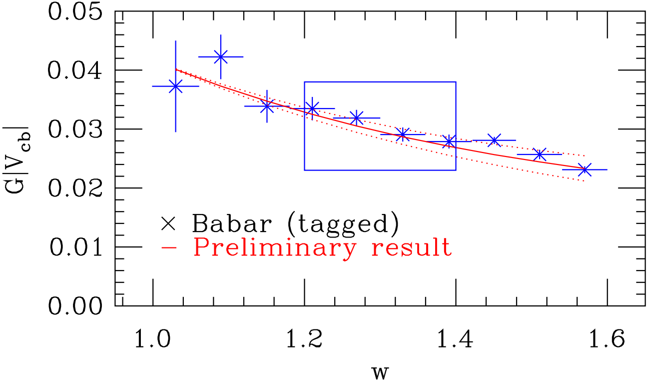
$$\mathcal{F}(1)_{\rm thy} = 0.908 \pm 0.017$$

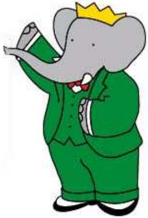
- This yields

$$|V_{cb}| = 39.54 \pm 0.50_{\rm expt} \pm 0.74_{\rm thy} \; .$$

- LHCb will reduce the experimental error significantly. Theory needs to keep up!

$|V_{cb}|$ from $B \to D\ell\nu$





- The exclusive $D\ell\nu$ channel can also be used.
- Here we look at the full form factor.
- The red lines give the theoretical prediction with error band.
- Theory has small errors at low recoil parameter w. Experiment has small errors at larger w.
- Ongoing project. Results will be reported soon.

Beyond the Standard Model: $B \to D\tau\nu$

- $B \to D\tau\nu$ *vs.* $B \to D\mu\nu$
- The ratio of the branching ratios

  $$R(D) = \frac{BR(B \to D\tau\nu)}{BR(B \to D\mu\nu)}$$

  is sensitive to new physics — *e.g.*, decays mediated by a charged Higgs boson.
- Previous Standard Model predictions for this ratio were done using approximate phenomenological models.
- Experimental measurements by the Babar Collaboration [arXiV:1205.5442] disagreed at 2.0σ with those predictions.
- A new lattice calculation by the FNAL/MILC [PRL **109**, 071802 (2012)] reduces this discrepancy to 1.7σ.
- The analogous decay of the D^* has a 2.7σ disagreement with approximate models.
- A lattice calculation for this one is in progress.

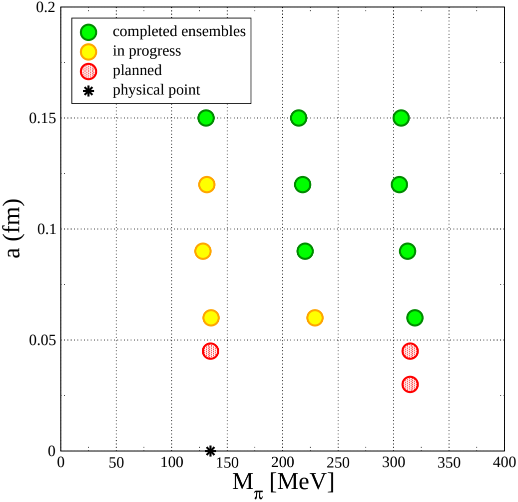
Computational campaigns – current practice

- Step 1. Generate gauge configuration files. *e.g.*, MILC HISQ ensembles.

$\approx a$ (fm)	m_l / m_s	$N_s^3 \times N_t$	$M_\pi L$	M_π (MeV)	N_{lats} (2012)	N_{lats} (2013)
0.15	1/5	$16^3 \times 48$	3.78	306.9(5)	1021	1021
0.15	1/10	$24^3 \times 48$	3.99	214.5(2)	1000	1000
0.15	1/27	$32^3 \times 48$	3.30	131.0(1)	1020	1020
0.12	1/5	$24^3 \times 64$	4.54	305.3(4)	1040	1040
0.12	1/10	$24^3 \times 64$	3.22	218.1(4)	1020	1020
0.12	1/10	$32^3 \times 64$	4.29	216.9(2)	1000	1000
0.12	1/10	$40^3 \times 64$	5.36	217.0(2)	1029	1029
0.12	1/27	$48^3 \times 64$	3.88	131.7(1)	840	1000
0.09	1/5	$32^3 \times 96$	4.50	312.7(6)	1011	1011
0.09	1/10	$48^3 \times 96$	4.71	220.3(2)	1000	1000
0.09	1/27	$64^3 \times 96$	3.66	128.2(1)	529	702
0.06	1/5	$48^3 \times 144$	4.51	319.3(5)	1000	1000
0.06	1/10	$64^3 \times 144$	4.25	229.2(4)	589	662
0.06	1/27	$96^3 \times 192$	3.95	135.5(2)	31	240
0.045	1/5	$64^3 \times 192$	4.50*	315*	0	51
0.045	1/27	$128^3 \times 256$	3.95*	135*	0	0
0.03	1/5	$96^3 \times 288$	4.50*	315*	0	0

Computational campaigns – current practice

- MILC HISQ ensembles. Graphical view.



Computational campaigns – current practice

- Step 1. Generate gauge configuration files.
  - Monte Carlo process, so need good statistics to reduce errors.
  - Need some two-dozen ensembles of typically 1000 gauge configurations each.
  - Lattice spacings (grid size) $0.045 - 0.15$ fm.
  - Box size $3 - 5$ fm on a side. (*cf.* proton size 1 fm).
  - Varying light quark masses with $m_{u,d}$ ranging down to physical values.
- Step 2. Calculate the necessary matrix elements with these configurations
- Cost: of the order 100 M core-hours per year in recent years.

Algorithmic Challenges

- Improved fermion formulations (*e.g.*, better discretizations of derivatives) reduce lattice artifacts.
- Dramatic improvements in the computational power of lattice QCD have come equally from algorithmic improvements and more powerful computers.

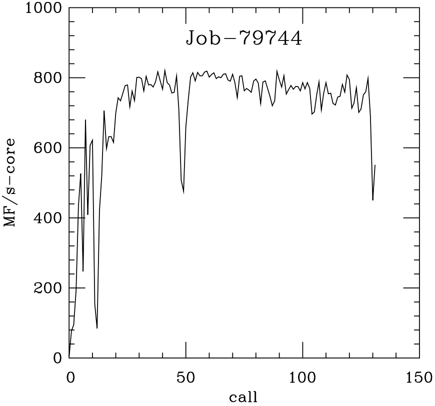
Algorithmic Challenges

- Greatest calculational cost: large sparse matrix problem $Ax = b$ (Dirac equation). Typically ill-conditioned.
- Nonstandard matrix format: So we must develop our own solvers.
- Start with standard methods: *e.g.*, conjugate gradient, biconjugate gradient
- Improvements
  - Multishift Krylov methods for multiple masses.
  - "Mixed" precision (*i.e.*, low precision preconditioners)
  - Deflation methods help for smaller problems (EigCG)
  - Multigrid methods help for larger problems
  - Additive-Schwartz domain-decomposition preconditioners.
- USQCD Collaboration: DOE SciDAC software effort plus SciDAC Institutes.

SciDAC
Scientific Discovery through
Advanced Computing

Contemporary computational challenges

- Jobs require up to several thousand processors.
- Network contention with other users could cause erratic performance



- Some parallel I/O issues. Performance of I/O systems have not scaled with system size.
- New, diverse architectures have us scrambling to reoptimize codes.

Looking ahead: computational challenges

- Scale disparities in our physics
  - More emphasis on b quark physics.
  - Small pion mass requires large box size (*e.g.*, $L = 5$ fm).
  - Large b quark mass requires small lattice spacing (*e.g.*, $a \leq 0.05$ fm) to reduce lattice artifacts.
  - Then number of lattice points should be greater than $L/a \approx 100$.
- We are currently experimenting with lattice sizes of $144^3 \times 288$ (250 GB gauge configuration files and 1 TB propagator files!)

Conclusions

- Thanks to improved computer resources and algorithms, lattice QCD has brought dramatic progress in high precision flavor physics.
- Further progress in searches for physics beyond the Standard Model requires a concerted effort by both experiment and theory.
- To meet the challenge requires continued improvement in resources and algorithms.
- We look forward to exciting discoveries to come.